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THE UNIVERSITY OF ALBERTA

THE USE OF HAND-HELD CALCULATORS IN GRADE 8 MATHEMATICS

BY

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A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled The Use of Hand-held Calculators in Grade 8 Mathematics submitted by Edward L. Klopoushak in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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Date October 12, 1978

TO EILEEN

Abstract

One purpose of this project was to study the effects on students, teachers, and the mathematics curriculum when eight students in a grade 8 mathematics classroom were provided with calculators. These students were taught the use of the calculator, were tested on the use, and were observed in the classroom. Parents' opinions on the use of calculators were included in the study. Twenty-six research questions were formulated.

Two grade 8 mathematics classrooms in two schools of the Edmonton Public School System were selected. Eight students from each school were provided with a Texas Instruments TI-30 calculator. Each group was taught the use of the calculator, in three forty-minute periods per week for twelve weeks. Students were taught general use of the calculator and were given exercises on rational numbers and on rate, ratio, and percent. These were topics in grade 8 mathematics at the time of the study. A tape recorder was used throughout every lesson to provide data from which many of the research questions were answered.

The researcher visited the classrooms for a total of twenty-two visits. The participant-observer model was used in this part of the study. The purpose of the visits was to note possible teacher adjustments to the presence of calculators. Students were observed to see how and when calculators were used, and the response of non-calculator students to the presence of calculators. The researcher participated by helping students to use the calculator. He asked calculator students to explain how they used the calculator. From these observations other research questions were answered.

Calculator Students were given various researcher-made tests, checks, and checkpoints as well as a standardized test on mathematical concepts and problem solving. Students were interviewed (checkpoints) six times in the project and once as a follow-up. A tape recorder used during class instruction and interviews provided data for some research questions.

A number of conclusions and interpretations were made. Generally teachers accept the presence of calculators in the hands of some students but they do not make efforts to provide special exercises or in other ways accommodate the calculator presence.

Low-achieving students can learn to do basic calculations and calculations involving direct key sequence. If a mathematical decision must be made then such students usually make many errors.

It was observed that three months is not sufficient time to produce good calculator usage. It takes much longer to produce desirable calculator techniques.

Student attitudes to mathematics did not alter significantly over the time of the project. Calculator and non-calculator students progressed approximately equally in knowledge of basic facts. In a rational number test calculator students did significantly better than non-calculator students but on rate, ratio, and percent the difference was not significant. Calculator students progressed as well as would be expected in mathematical concepts and in problem solving.

Parents are generally supportive of the use of calculators including using them in tests. They recognize their presence in society but also feel that basic mathematical skills will decline.

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CHAPTER I

THE PROBLEM

> With the decrease in cost of the minicalculator, its accessibility to students at all levels is increasing rapidly. Mathematics teachers should recognize the potential contribution of this calculator as a valuable instructional aid. In the classroom, the minicalculator should be used in imaginative ways to reinforce learning and to motivate the learner as he becomes proficient in mathematics. (NCTM Instructional Affairs Committee, 1976, p. 92).

With the above position statement the Instructional Affairs Committee of the National Council of Teachers of Mathematics (NCTM) introduced its report on Minicalculators in Schools made to the Board of Directors of NCTM. This statement is accepted as the official position of the NCTM. The mini- or hand-held calculator is everywhere. Its impact on education in general and on mathematics education in particular is widely recognized by prominent writers and researchers in current journals and books. Caravella (1977) introduces his book with a chapter entitled, "The Age of the Minicalculator" which he begins with:

> We have entered the age of the minicalculator. Indeed history may well consider the technical development of minicalculators in the United States as a bicentennial contribution to world progress. (p. 13).

The impact of the minicalculator can be gauged somewhat by the quality and quantity of activity it has engendered. A number of persons have developed activities that would be suitable for classroom use at various grade levels. Many informal projects involving the use of minicalculators have been conducted by teachers in many parts of the United States, Canada and in several European countries. More formal research projects, mostly at upper elementary or higher grades, have been carried out and reported. These have been either graduate school research projects or other research conducted by university faculty researchers. In addition a number of conferences have been held specifically for the purpose of drawing together the collective opinions of many different individuals or groups. One such conference was sponsored jointly by the National Institute of Education and the National Science Foundation of the Government of the United States. (Report of the Conference on Needed Research and Development on Hand-held Calculators in School Mathematics, 1976). Another important project under Marilyn N. Suydam as director and supported by the National Science Foundation was carried out in 1975 with the report produced early in 1976. (Suydam, 1976).

Since these two major conferences the amount of activity has increased so that every conference

(NCTM-sponsored and others) on mathematics education has a number of sessions devoted to some aspect of the use of calculators[1] in the classroom. Almost every issue of the major mathematics education journals carries one or more articles dealing with a phase of the use of calculators in the classroom.

With the advent of a tool that is viewed by many educators as potentially having a tremendous impact on mathematics education, it is not surprising that a great deal of research is needed to assess the proper utilization of calculators in the total school mathematics program.

It is under these circumstances that the research project described in this document was undertaken.

Statement of the Problem

General

What effects are produced when the hand-held calculator is present in the hands of some students in grade 8 mathematics classrooms?

---

[1] See Appendix A for Definitions. The word "calculator" is used to refer to the mini- or hand-held calculator.

Specific

Eight students from each of two grade 8 mathematics classrooms (a total of sixteen students) were provided with calculators and given instructions in their use.

The following questions form the basis of the study:

What effects are produced on the students with the calculators?

What are the effects on the mathematics curriculum as it relates to the students with calculators?

What are the effects on the teachers and on school policy?

What are parents' opinions on the use of calculators by their children in grade 8?

Theoretical Background of the Study

Easley (1977) documents seven models used in research on teaching and learning. These include combinatorial models, sampling models, cybernetic models, game models, critical thinking models, ordinary language analysis models, and dynamic structural models. The models encompass a broad range of mathematical models from multidimensional (regression, factor analysis) to direct analysis of tapes of interviews. The last model, dynamic structural, applies in assessing

"... the structures (dynamic forms) of the ideas and feelings underlying (assimilating) human behavior." (p. 323). Although the study undertaken by the researcher does not fall wholly within the dynamic structural model, it does make use of some of the techniques suggested under this model.

A relatively small sample (sixteen students) was studied much more intensely than would be possible or desirable in a more traditional experimental-control group design. Through personal interviews, calculator group assessment, in-class observations, and assistance from teachers through informal discussions, each student was assessed on an individual basis with respect to the various aspects considered significant in this study. Therefore much of the data reported are from individual students and not from general whole-class measurements. It is assumed that this approach provides insight into how students work with calculators and what thought processes calculators evoke. In that respect the study follows the format used by Erlwanger (1974). As he points out:

> The view adopted here ... is that in the course of learning mathematics a child develops his own ideas, views and beliefs about mathematics which can be represented as his conception of mathematics. This concept of mathematics may be regarded as a developing conceptual system of interrelated ideas, beliefs, emotions, and views concerning mathematics and learning mathematics that directs and controls his mathematical behavior, how he

> learns and what he understands. From this point of view, a child's observable mathematical behavior may only be interpreted and explained to the extent that his underlying conception is understood. (p. 6).

More recently a more sociologically-based approach to study of education has been advocated by several writers. Disappointment with many traditional studies has been voiced repeatedly. Higginson (1977) in describing a speech by Bronfenbrenner in 1976, pointed out that Bronfenbrenner, "... implies, among other things, that the classic, statistically-oriented model of research has been a failure in the field of education." (p. 50). In elaborating on this theme, Higginson (1977) said:

> In an attempt to appear scientific, educational researchers have accepted an antiquated vision of simple cause and effect linked to primitive classifications and sophisticated statistical techniques. (p. 50).

He goes on to point out that this type of research model is a poor one upon which to make conclusions about classes, teachers or treatments when he adds:

> Stated differently this criticism says that in a typical classroom one has not a 'two-body problem' with one T (teacher) and one C (class), but a 'thirty-six body problem' with one T, and thirty-five C (children). The latter is a very much more complex situation than the former; it has, however, the merit of reflecting the reality of the classroom. (Higginson, 1977a, p. 50).

Much support for forms of research that are more pupil-intensive have been advocated by other writers -- Higginson (1977b), Lankford (1974), Wilson (1975), Bauersfeld (1976), and Rappaport (1977). Further support emanated from a conference on hand-held calculators sponsored jointly by the United States National Institute of Education and the National Science Foundation. In one of the recommendations the report said, "Methods can range from clinical trials to full scale randomized intervention experiments, ..." (Report of the Conference on Needed Research and Development on Hand-held Calculators in School Mathematics, p. 17).

In this study the methodology is taken also from the somewhat more recent participant-observer strategy. West (1977) writes by way of opening remarks in an article of participant-observer techniques, "There has been a growing disenchantment in the last few years with the process and results of traditional educational research and development approaches." (p. 50). He also draws attention to the lack of research centering on behavior in the classroom when he says, "It is thus strikingly noticeable how few research studies have attempted to obtain behavioral data on classrooms, the central setting of most of our educational concerns." (West, 1977, p. 56).

Because of these shortcomings West (1977) advocates alternative methodology to be applied in

Canadian educational research, which he claims is in its infancy. The alternative is that of the participant-observer, of which West says, "Another research tradition, developed largely within sociology and anthropology, explicitly offers better hope for the discovery and development of the new types of knowledge demanded in such an unexplored area as the classroom." (p. 59). In this type of research the researcher gives great emphasis to behavior of people in particular situations; he gets close to the subjects in as full and complete a way as possible. The type of research is usually conducted over some time and this is necessary since, "... humans create their own social environments, and change them." (West, 1977, p. 60).

Both roles of participant and observer are important. As a participant the researcher becomes somewhat as socialized as his subjects and in this way he gets closer to the "feel" of the situation; he becomes like one of the subjects. However this socialization can go too far and his effectiveness as a researcher may be in jeopardy. Therefore the researcher attempts to "... cultivate his awareness without becoming as socialized as the subjects." (West, 1977, p. 61). He does this by alternating between participant and observer.

By being in the classroom the researcher observes behavior where it actually takes place, in the complex classroom world. His techniques are not all structured so that there is a good chance of making discoveries which could not have been foreseen. This must therefore be essentially an unstructured research or at least one with a low initial structure. It is true that as the study progresses, additional research questions may be formed in view of unforeseen discoveries, or existing research questions may be altered.

The study of hand-held calculators in the hands of some grade 8 students is an attempt to consider individual children in their "natural" school environment. That is, the study did not generate exclusively data on groups (means, standard deviations, variances, correlation coefficients) and then try to draw conclusions about the groups. Rather it attempted to describe behavior of children under the circumstances stated above. Some statistical data, however, are included in the study.

Need for the Study

There is no doubt that the hand-held or minicalculator is having and will continue to have a dramatic impact on mathematics education. The decrease in cost (about $20 for a "scientific" calculator or as little as $5 for a basic four-function calculator with memory) has placed this powerful tool in millions of hands across North America. One projection says that by 1980 there will be about 80 million calculators in use in the United States and that after 1980 about 20 million will be purchased annually. (Report of the Conference on Needed Research and Development on Hand-held Calculators in School Mathematics, 1976, p. 1). These figures compare to 70 million homes in the United States (figures for 1974) or 132 million telephones (95 million of them in homes). (Report of the Conference ..., p. 1).

Not only has the hand-held calculator appeared on the North American scene in general, but quite naturally it has made its appearance on the educational scene. How to accommodate this fascinating aid or tool into the mainstream of education has become the subject of much debate and more recently, the topic of several research activities.

In June 1976 the National Institute of Education and the National Science Foundation of the United States jointly sponsored a conference in Arlington, Virginia (the report is quoted above). To this conference were invited selected specialists. The letter of invitation asked the participants to, "produce a ... planning document that will provide a well-defined framework for future research and development efforts [that may be used] as a guide to program planning." (Report of the Conference, ..., p. v). The calling of the conference in itself attests to the need for research in this area.

In May 1974 the Conference Board of the Mathematical Sciences of the United States appointed a National Advisory Committee on Mathematical Education (NACOME). Their report, Overview and Analysis of School Mathematics K-12, commonly referred to as the NACOME Report, devotes several pages specifically to the implications of the presence of calculators on mathematics in the school. (Hill, 1975). Once again this is evidence for the need for further research into the implications for education of the existence of calculators in the educational setting. The Report makes the following statement, "Despite the obvious promise of calculators for enriching mathematics instruction, important questions of their optimal use must be investigated by thorough research." (Hill, 1975, p. 42). The Report goes on to list several areas for research, among them:

> When and how should calculator use be introduced so that it does not block needed student understanding and skill in arithmetic operations and algorithms?
>
> ....

> What special types of curriculum materials are needed to exploit the classroom impact of calculators?
>
> How does calculator availability affect instructional emphasis, curriculum organization, and student learning styles in higher level secondary mathematics subjects like algebra, geometry, trigonometry, and calculus? (Hill, 1975, p. 42).

The United States National Institute of Education Conference on Basic Mathematical Skills and Learning held in Euclid, Ohio in October, 1975 resulted in a two volume report containing position papers (Volume I) and committee reports (Volume II). Not surprisingly a large number of references were made to the impact of calculators on future mathematics in the schools.

The nonthematic chapter of the 1977 Yearbook of the National Council of Teachers of Mathematics is on hand-held calculators. This is another indication of the concern which the calculator has evoked. In this chapter the call for research is clear, as evidenced by the following statements taken from several authors as points upon which there is agreement:

> 1. the calculator cannot be ignored: its use in the schools must be carefully explored;
> 2. the focus of attention must be on how the calculator can best be used to develop and reinforce mathematical skills and ideas;
> 3. studies must be made of the uses of mathematics needed by children and by adults, given widespread use of the calculator in our society;
> 4. research and curriculum development must proceed hand in hand;

> 5. although some attention must be given to immediate concerns, this must not preclude long-range planning for mathematics curricula and instructional practices that incorporate optimal use of the calculator. (Bell and others, 1977, p. 239).

The need for further research in the application of the calculator has been voiced repeatedly by many people. In addition to those cited, Gawronski and Coblentz (1976) expressed this need when they said, "There are some researchable questions to be examined as well as curriculum uses to be identified." (p. 512). Further support for needed research was expressed by Hilton and Rising (1975) at the Euclid, Ohio Conference on Basic Mathematical Skills and Learning. They call for research into all aspects of the use of calculators. (p. 39). They state that some calculator manufacturers are publishing their own curriculum materials and the quality of these will likely lead to a great deal of research.

The above conferences clearly document the need for a variety of research efforts with reference to the impact of calculators in the schools. Further documentation is found in the review of the literature.

Outline of the Study

Chapter II contains a review of the literature as well as the framework and the research questions for the study. Chapter III describes the selection of students for the sample, the instructional pattern, the instruments used and the structure of the analysis of the results. In Chapter IV the research questions are answered in detail using the data collected. Three cases are described extensively in Chapter V. These are respectively a high academic achiever, an average achiever, and a low achiever. The chapter also compares the three students on their ability on the calculator. Chapter VI contains the conclusions and makes some suggestions for further studies.

CHAPTER II

RELATED STUDIES AND RESEARCH QUESTIONS

The purpose of this chapter is to build a base for a framework of research questions. What follows is a discussion of suggestions for calculator use and its effects. The discussion proceeds from consideration of informed sources, to the deliberations of organized bodies, and finally to the formalized results of research studies. Findings from these sources are molded into a framework of questions relating to teachers, to students, to the mathematics curriculum, and to parents.

Review of the Literature

Suggestions for Calculator Activities

Much of the calculator activity reflected in the literature is in articles expressing opinions and giving ideas about how the calculator could be used and what effects it might have. Suggestions are made for various games, activities, or puzzles to be tried in the classroom (some examples are Hawthorne (1973), Kahn (1976), Hobbs and Burris (1978), Lappan and Winter (1978), Drake (1978), and Waits (1978)).

Hopkins (1976) suggests that we make the fullest use of the calculator in all grades. He feels that the loss in ability to do traditional algorithms would be more than offset by the gains in such things as working with very large numbers easily and accurately, doing complicated work problems and studying theory of arithmetic. Immerzeel (1976b) in an article in the Arithmetic Teacher makes a number of suggestions about how calculators could be incorporated into the regular classroom even on a limited basis, that is, where not every student has one but rather where only one might be provided in the interest corner.

Immerzeel (1976a) also prepared a book giving some ideas and suggestions for using the calculator in the classroom. He suggests that calculators, "... should not be used until students have a firm grasp on the concept of number and of the basic operations." (p. 4). In addition to stressing that basic computation skills be developed, Immerzeel (1976a) feels the calculator forces students to sharpen estimation and recording skills:

> It is easy to make mistakes when using the calculator. To help eliminate these mistakes, children should be taught to always visually check each entry to be sure it is correct and to estimate the final answer to catch obvious errors. (p. 14).

In his book Immerzeel provides many examples to help students develop skills such as estimation.

Jacobsen also makes a case, and offers suggestions, related to the use of calculators. As does Immerzeel, Jacobsen (1977) calls for a change in emphasis to include rounding, estimation by rounding, addition and subtraction of decimals, and multiplication of decimals to two or three significant figures. He feels that students can compute but can not solve problems. Use of calculators would enable the emphasis to be shifted from the calculation to the analysis of the problems. Much the same points are made by Kaufman. (Kaufman and others, 1975).

Some suggestions for use of the calculator in the schools are made by Kahn (1976) and Machlowitz (1976) while Maor (1976) makes a number of suggestions for using the calculator in more advanced mathematics such as the algorithms for evaluating $ab + cd$, $\frac{a}{b} + \frac{c}{d}$, $a + b^2$, and ab^2.

Many of these kinds of activities were used in the project and are reflected in the research questions which follow in this chapter.

Recommendations of Organizations

Because of the profound impact of the calculator on our society the National Council of Teachers of Mathematics, headquarters in Washington, D.C., through its Instructional Affairs Committee has declared a policy concerning the use of calculators. The quotation at the beginning of this paper is the introduction to their policy statement. The committee makes a number of suggestions about how it believes the calculator could be used (the following omits examples and further explanations which follow each use given):

> The minicalculator can be used to encourage students to be inquisitive and creative as they experiment with mathematical ideas.
>
> The minicalculator can be used to assist the individual to become a wiser consumer.
>
> The minicalculator can be used to reinforce the learning of the basic number facts and properties in addition, subtraction, multiplication and division.
>
> The minicalculator can be used to develop the understanding of computational algorithms by repeated operations.
>
> The minicalculator can be used to serve as a flexible "answer key" to verify the results of computation.
>
> The minicalculator can be used as a resource tool that promotes student independence in problem solving.
>
> The minicalculator can be used to solve problems that previously had been too time-consuming or impractical to be done with paper and pencil.
>
> The minicalculator can be used to formulate generalizations from patterns of numbers that are displayed.
>
> The minicalculator can be used to decrease the time needed to solve difficult computations. (NCTM Instructional Affairs Committee, 1976, p. 92-94).

The NACOME Report produced by the National Advisory Committee on Mathematics Education of the National Council of Teachers of Mathematics, makes a recommendation dealing specifically with the use of calculators. Its implications are obvious:

> . . . that beginning no later than the eighth grade, a calculator should be available for each mathematics student during each mathematics class. Each student should be permitted to use the calculator during all of his or her mathematical work including tests. (Hill, 1975, p. 138).

One frequently encounters references stating that slow students should not use calculators since they have not yet mastered the basics (arithmetic computations). This argument is applied even if (or maybe especially if) students are in the upper grades. It is interesting to note that the NACOME Report on this point concerning students at the end of the eighth grade says:

> We feel that providing such students with electronic calculators to meet their arithmetic needs and allowing them to proceed to other mathematical experience in appropriately designed curricula is the wisest policy. (Hill, 1975, p. 25).

This report makes a number of relevant observations also stressed by others. One such observation is that elementary schools likely will work with decimals much earlier while work with common fractions will be delayed and deemphasized. Calculators will very early lead students to explore and study negative integers, exponents, square roots, scientific notation, large numbers; computational ability will probably come long before a careful conceptual development of many of these topics. Mathematics proficiency was assumed to be needed before conceptual study and application. This meant that poor students rarely if ever got to explore more exciting topics in mathematics because they were always blocked by their inability to get beyond the computation skill. With calculators such students, claims this report, should be able to get beyond the computational level and delve into aspects of probability, statistics, functions, graphs or coordinate geometry. The calculator however will not remove the necessity to analyze problems and interpret numerical results. With decreased time devoted to the computational aspects of problem solving, students should be free to develop appropriate skills related to analysis and interpretation. Finally, the report draws attention to the necessity for a reevaluation of tests. If calculation remains a prime goal of testing the calculator will invalidate present standards.

In another part of the National Institute of Education Report of the Conference on Basic Skills and Learning, attention is once again drawn, under a discussion of Basic Goals, to the impact of calculators. Comment is made on certain topics now emphasized in arithmetic:

> The time we currently spend teaching elaborate long division problems and complicated lowest common denomination fraction problems -- often with little success -- could be better spent on more interesting, rewarding, and motivating topics. (Report of the Working Group on Goals for Basic Mathematical Skills and Learning, 1975, p. 18).

The report also calls for a change in emphasis from endless and mindless drill in computation to the mathematical principles and concepts underlying the computation algorithms. The report calls for learning processes of computation together with skills in estimation and approximation as readiness for future learning.

The NACOME Report also makes several references to required research in the area of calculator usage in education. In addition to previous excerpts quoted the report states:

> Research is urgently needed concerning the uses of computing and calculating instruments in curriculum at all levels and their relationship to a broad array of instructional levels. (Hill, 1975, p. 143).

Later the report suggests that, ". . . there are particular areas considered in this report where new curricular organizations, instructional materials, and courses are of urgent concern." (Hill, 1975, p. 145). One place where it is suggested that instructional materials at all levels are needed is in the use of calculators.

The Report of the National Institute of Education

Conference on Basic Mathematical Skills and Learning also draws attention to the need for research, "The whole issue of the effect of the calculator on the teaching of arithmetic is a very complex one which deserves considerable investigation and consideration." (Report of the Working Group on Goals for Basic Mathematical Skills and Learning, 1975, p. 18). The same report suggests that students must learn single-digit number facts, including the multiplication table. In addition they should be fluent in simple types of computation. However,

> Exactly how much, between this "bare bones" minimum and the amount of computation that is currently being taught, is a question that needs further study and far more discussion among a broader base of people. (Report of the Working Group on Goals, 1975, p. 18).

Another document dealing solely with needed research on the use of calculators is one prepared jointly by the National Science Foundation and the National Institute of Education, both of Washington, D.C. (Report of the Conference on Needed Research and Development on Hand-held Calculators on School Mathematics, 1976). The report claims that educators should not default on their obligations to become fully involved in research and development in calculator use, thereby allowing manufacturers and publishers to make many of the crucial decisions. In speaking of the potential impact of calculators the report states, "Calculators and their successors [meaning, eventually, very cheap computers] may eventually influence the entire school mathematics curriculum and perhaps school curriculum more generally." (Report of the Conference on Needed Research . . . , 1976, p. 10).

The report makes a total of twenty-two recommendations grouped into five broad categories: development of an information base, development of curriculum materials (short-term and long-range), research and evaluation, teacher education and dissemination. Some recommendations follow, illustrating the concerns expressed at this conference:

> Recommendation 1. Existing mathematics curriculum materials should be intensively and critically analyzed to determine whether they can be adapted for use with calculators.
>
> Recommendation 6. Materials should be developed to exploit the calculator as a teaching tool at every point in the curriculum to test a variety of ideas and possibilities pending emergence of calculator-integrated curriculums.
>
> Recommendation 7. Research and development should be undertaken with respect to algorithmic processes in school mathematics, especially those in arithmetic and algebra and perhaps in trigonometry.
>
> Recommendation 10. Curriculum materials should be developed and tested on new topics that are not now in the curriculum but that calculators may make feasible. The possibilities of adapting "advanced" topics to earlier levels should also be explored.

> Recommendation 13. Sampling and research methodology should be appropriate to the type and scale of development. Methods can range from clinical trials to full-scale randomized intervention experiments, and from research focused on special subsets of the school population (e.g., remedial, handicapped, gifted) to that focused on a more general population.
>
> Recommendation 17. Every effort should be made to assess any changes in the amount of time spent in mathematics instruction and the distribution of that time as calculators become an integral part of the school experience. (Report of the Conference on Needed Research . . . , 1976, p. 17).

One of the working groups of the National Institute of Education Conference on Basic Mathematical Skills and Learning made reference to calculators and some needed research in the following areas:

> (1) alternative sequences for elementary instruction in arithmetic; (2) uses of the calculator as an aid and stimulus for arithmetic instruction; (3) impact of calculators availability on problem-solving instruction; and (4) relative importance of various familiar fraction concepts in an environment of calculators (to include investigation of curriculum topics in later courses such as algebra where the field properties of "fractions" are used. (Report of the Working Group on Curriculum Development and Implementation, 1975, p. 11).

Another extensive report dealing with calculators resulted from a National Science Foundation (United States) project carried out in May to July of 1975 with the final report published in February 1976. The project included collection of information by questionnaire, compilation and review of literature from many sources, and meetings which included presentations, workshops, discussions and review of manufacturers' displays. The intent of the project was:

> (1) To collect information regarding the use or non-use of calculators, and to form a list of the reasons why educators and others believed that the calculator should be used in schools or why the calculator should be banned in schools.
> (2) To analyze the arguments reported by those questioned and in the literature, in order to determine the potential impact or lack of impact of the calculator on the curriculum; and
> (3) To develop a critical analysis of what has and has not been done with calculators at pre-college levels, what knowledge is or is not available about them, and what implications this has for education at the pre-college level. (Suydam, 1976, p. 2).

The above report (Suydam) contains several references to the lack of adequately-researched studies on the impact of calculators. Bearing in mind that the project dates from mid-1975, one must assume that such statements would not apply in the same way today. However, in spite of the activities there is still a large amount of study necessary in order that calculators be integrated into the mainstream of our educational system.

Usiskin and Bell (1976) prepared a position paper which became Appendix F to the Suydam report. These authors, in addition to describing the ubiquitous presence of calculators and presenting a historical perception of calculation and analyzing calculation requirements of two widely-authorized elementary arithmetic series, also spent a major portion of their paper documenting changes in curricular emphases which are suggested to them by calculators. They make the point that calculators should not replace all paper and pencil algorithms but suggest that questions such as 346.25 X 18.97 have no real world applications and probably are best left to the calculator should the need arise for an evaluation of such a computation. One of the strengths of this paper is that a specific list of objectives is given for a calculator-oriented program. There is an obvious stress upon rounding, estimation by rounding, work with decimals and conversions of decimals to fractions and vice versa, "The use of simple computations to help estimate answers to more complicated computations must be given strong emphasis." (Usiskin and Bell, 1976, p. 22 of Appendix F).

In discussing the future of fractions, Usiskin and Bell see several important roles for fractions and these will continue to need attention (for example, in many formulas in sciences, fractions make for an easier grasp of the meaning of the formula). In situations such as $ax = b$ it is more appropriate to express the solution as $x = \frac{b}{a}$. The authors point out that no matter how much work is retained on fractions [or whether teachers will insist that calculations with fractions be solved with fraction solutions rather than permit decimal notation], it is a fact that the calculator will make most of the calculations trivial. (See Appendix H for a sample addition question using a calculator). Usiskin and Bell make three specific suggestions for work with fractions where calculators are used:

> 1. Less emphasis should be placed upon arithmetic operations with fractions.
> 2. More emphasis should be placed upon symbolic manipulations with fractions.
> 3. Continued emphasis is needed on the meaning of fractions and work with equivalent fractions. (Usiskin and Bell, 1976, p. 29 of Appendix F).

Formal Research Findings

Even though formal research involving calculators is not yet extensive a number of dissertations and other research projects have been carried out. Generally studies involved an experimental and a control group and lasted only several weeks with assessments made through pre- and posttests as well as attitude scales or opinionaires (for parents, for instance). It appears that no significant differences were found more often than otherwise. Allen (1976) taught a twenty-five-day unit on decimal algorithms and the metric system. The study involved four sixth grade classes who used calculators and two who did not. She found no significant differences of the posttest but the paper and pencil group scored significantly higher on the retention test. Her conclusion was that using pencil and paper only was more effective than using hand-held calculators for the retention of concepts and skills of decimal algorithms and metric units. Anderson (1977) studied the effect of using calculators on achievement and attitudes of seventh grade students. Although students using calculators showed improved attitudes to mathematics they did not change in achievement, understanding mathematical concepts or in computation skill. In problem solving, students with calculators could solve problems at twice the rate of students not using calculators.

Most studies reported no significant differences in some aspects of the studies and significant differences in others. Fischman (1976) in a project with New York City high school students in business arithmetic found no significant differences in attitudes but the calculator group scored significantly higher on a test of arithmetic skills. She found no significant difference in understanding business arithmetic concepts between the students who did the unit using the calculator and those who did not.

Hutton (1977) found no significant differences in attitude or achievement among three groups of ninth grade algebra students. The groups included the C or control group; the E_t group, which used calculators and these the teachers incorporated into the teaching using special notes; and the E_s group, which used calculators but the teachers did not incorporate these into their teaching. Only one teacher (of the four participating in the study) had a significant difference in attitude assessment for the calculator group over the non-calculator group. Although statistically the differences were not significant, Hutton reports that all teachers and students were enthusiastic about the use of calculators and felt they were an aid to learning and made the lessons more enjoyable.

Jamski (1977) found a significant difference favoring the calculator group on an immediate posttest but no significant differences on a retention test. His study was with grade 7 students on the subject of Rational Number-Decimal-Percent conversion. In addition to the statistical aspects of the study he reports that (1) students enjoy using the calculator because of the novelty of employing a machine, (2) students are selective about when to use the device, and (3) calculators are not a panacea to all problems of mathematics instruction.

Jones (1976) found that sixth grade students using calculators gained significantly higher in total mathematics achievement, computation and concepts; she also found a significant difference between female and male students (favoring females) on concept scores. However, no significant differences were found between males and females in total mathematics achievement, computation, attitudes or self-concept; no significant differences were found among all students in attitudes or in self-concept. She concluded that using calculators was more effective in total mathematics achievement, computation, and concepts than using only pencil and paper.

Muzeroll (1976) worked with seventh graders and found no significant differences in mathematics attitude or achievement between groups. He did find an overall significant decline in attitudes towards mathematics from the end of grade 6 to the end of grade 7 for all students. Lenhard (1977) analyzed eight tests taken by students in grades 7 to 12 and found no differences in performance between students using calculators and those not using calculators, on test scores, concept and computation errors, attitudes, time and rank. Sutherlin (1977) studied decimal estimation skills with eight grade 5 and 6 classes, four using calculators and four not using them. He found no significant differences in estimation skills.

Shuch (1976) compared the achievement of community college business mathematics students using calculating machines with students using traditional pencil and paper methods of computation. He also compared their achievement with respect to critical thinking ability. He rejected the null hypothesis that using the calculators will improve students' ability with arithmetic fundamentals, arithmetic problem-solving ability, or critical thinking ability. In studying ninth grade and college students Zepp (1976) investigated the interaction of the calculator with specific reasoning ability as manifested in proportional thinking. He divided the grade nine students as well as the college students into high and low ability groups forming four different groups. Half of each group were given calculators while the others used only pencil and paper for computation. The differences between posttest means were significant between the ninth grade groups but not between the college groups. He found that students using calculators did not perform better than the non-calculator students. He concluded that difficulties with proportional reasoning ability can not be explained by assuming that computational deficiencies are the main stumbling block.

Quinn (1976) used programmable calculators with eighth and ninth grade students and assessed attitude and achievement. He found no significant differences between calculator users and non-calculator users on achievement in algebra. In addition he concluded that at the eighth grade level, data provide no indication that improvement occurs or is superior on attitude scales; at the ninth grade the indication is that the programmable calculator could be helpful in maintaining and improving some aspects of student attitude when used in a standard algebra program.

Ladd (1974), in studying attitude and achievement of ninth grade low achievers in mathematics, found some significant improvement in both but this occurred in a very limited setting -- low achievers, short lessons, problems from local businesses, experienced teachers, and small classes. He also found, however, that adding electric calculators to the course of instruction in the above setting did not increase nor decrease the improvement in attitude or in achievement.

Borden (1977) studied sixth graders' learning and attitudes in concepts and skills in decimal fractions. He found (1) a significant gain, favoring the calculator group, in concepts of decimals as demonstrated on a paper and pencil test not using the calculator, and (2) students not using the calculator had a significant negative attitude to mathematics.

Miller (1977) found that a low (on the basis of prerequisite skills) calculator group scored significantly higher than the low control group on posttests in prerequisite skills and in division. Nelson (1976) found that gains in basic computational skills and attitudes towards mathematics were significantly better for students using calculators.

Spencer (1975) found that the fifth grade calculator group scored significantly higher on the reasoning test and the sixth grade calculator group did the same on the computation test and on the total arithmetic test. Vaughn (1977) found, in a study of ninth grade general mathematics classes, that the calculator group scored significantly higher on an achievement test. He noted no differences in attitudes or on retention of skills. Wajech (1976) reported similar results: significantly higher scores on a standardized computation test but no significant differences in attitudes. Whitaker (1977) studied first graders and found that calculators aided students in non-timed computations and in solving verbal problems. The calculator group, who used calculators to check their own daily work sheets, displayed smaller gains in mathematics conceptualization. No significant differences were found in timed computations, total achievement gain or attitude gain.

Aldridge (1977) used pre- and posttests with remedial students. She found a significant difference favoring the non-calculator group on the basic skills test but no significant differences when the test was compared by grade level. Nichols (1976) studied the effect of electronic calculators in a basic mathematics course for college students. He found very little difference in achievement, a better attitude toward mathematics for the calculator group but not significantly so. Generally he found that the use of calculators is of significantly more benefit in improving attitude and achievement for students of higher aptitude in mathematics than for students of lower aptitude.

O'Loughlin (1975) used programmable calculators in a first year university calculus course. As a result of the study he feels that the use of such a calculator allows introduction of topics not found normally in a beginner's calculus course but also permits coverage of the traditional material. The treatment group did significantly better in certain topics of calculus although there was no difference in attitude in the two groups. However the treatment group did prefer a computational to a theoretical approach and indicated a strong desire for continuing use of a mini-computer (programmable calculator) in calculus.

An interesting study involving algorithms for operations on positive rational numbers for low-achieving ninth grade mathematics students was carried out by Gaslin (1975). This study was conducted in the fall of 1971. He used two algorithm sets: one was a conventional algorithm method of operations on rational numbers using the usual text approach, a second was an alternative algorithm in which each fraction was converted to a decimal on the calculator (truncated to thousandths) followed by the operation. He had three treatment groups: one used the conventional method and did not use calculators, a second used the same method but used calculators while the third did the alternative method using calculators. He stated that from his results it appears that the alternative algorithm with the calculator is a viable alternative to the conventional algorithm for low achievers. However, if the goal of instruction is learning of the conventional algorithm then the calculator did not significantly affect performance. He found no differences in development of positive attitudes. In general Gaslin observed that the alternative algorithm provided the ninth grade low achievers with a more efficient method in terms of rate of mastery and retention.

Framework and Questions for the Study

The study was of the type utilizing process descriptions based on classroom situations, and interactive clinical interviews. There were three basic dimensions:

1. classroom teaching and testing of two calculator groups,

2. interactive clinical interviews with sixteen students and,

3. classroom observation and testing (participant-observer model) of two classes from which the calculator students were selected.

Based on the literature and the style of the study, the following research questions appeared to be appropriate:

Research Questions Related to Teachers

Q1--How do teachers adapt to the presence of calculators in the hands of some of their students in a grade 8 mathematics class?

Q2--To what extent do teachers become knowledgeable about the possible role of calculators in a grade 8 mathematics classroom?

Q3--What is the attitude of the teachers about the possible use of calculators in grade 8 mathematics?

Research Questions Related to Regular Student Learning

Q4--To what extent do grade 8 mathematics students learn to use a calculator effectively to do basic mathematics calculations?

Q5--Will the use of calculators improve the students' ability to estimate answers to questions involving basic operations?

Q6--Can grade 8 students effectively use calculators to assist with calculations required for traditional algorithms associated with operations with rational numbers?

Q7--Will the calculator enable grade 8 students to cope with more complex numbers (for example, $12\frac{13}{17}$ as compared to $8\frac{1}{2}$) in doing rational number algorithms?

Q8--Will students prefer working with rational numbers in decimal form for various operations rather than in fraction form?

Research Questions Related to Student Learning Beyond Standard Curriculum

Q9--Because of their ability to use the calculator, will grade 8 students go beyond curriculum requirements in mathematics?

Q10--What aspects of Rational Numbers and Rate, Ratio, and Percent will be trivial for students knowledgeable in the use of calculators?

Q11--Because of ability with calculators will students pursue various topics not in the standard curriculum upon the suggestion of the researcher?

Q12--Will students show interest in following up on mathematics problems posed by the researcher and develop their own algorithms in solving these problems?

Research Question Related to High-achieving Students

Q13--Will high-achieving students be much more active in exploring and going beyond basic requirements than will low-achieving students?

Research Question Related to Low-achieving Students

Q14--Will low-achieving students be able to do basic calculations efficiently using the calculator; will they limit their calculator activities to minimum curriculum requirements?

Research Questions Related to the General Use of the Calculator

Q15--What are the necessary techniques for effective calculator use?

Q16--How well do calculator students use the calculator?

Research Questions Related to Student Attitude and Interest

Q17--How does the attitude of the calculator students change over the period of the project?

Q18--Will student interest in the use of the calculator be high? Will interest continue without further stimulation by the researcher at the conclusion of the project?

Research Question Related to the Curriculum

Q19--Which topics of mathematics lend themselves to calculator application and which do not?

Research Question Related to Parent Attitudes

Q20--What are the attitudes of the parents of calculator users towards the use of calculators?

Research Questions Related to Further Changes in Students

Q21--What changes occur in the calculator students' knowledge of mathematics concepts and in problem solving ability as measured by the Canadian Test of Basic Skills?

Q22--How did the calculator students change in basic knowledge of computational facts (addition, subtraction, multiplication, division)?

Q23--How did the calculator students change in knowledge of Rational Numbers?

Q24--How did the calculator students change in their knowledge of Rate, Ratio, and Percent?

Research Questions Added during the Project

Q25--What general mathematical knowledge or skills appear important in calculator applications?

Q26--What might a teacher do to incorporate calculators into regular classroom mathematical lessons?

CHAPTER III

THE SAMPLE, THE INSTRUMENTS, AND THE ANALYSIS

Grade and Topic Selection

A great deal of controversy surrounds the use of calculators at all levels up to and including junior high school. However, there is much more opposition about the possible use of calculators up to grade 6 than there is in grades 7, 8, or 9. For this reason the junior high school level was selected for study. Grade 8 was chosen because these students are adjusted to the junior high school setting and because school authorities or individual teachers are more resistant to wide use of calculators in grade 7 than in grade 8.

The time of the school year for which the project was set meant that grade 8 would be working on the unit on Rational Numbers and possibly on the unit on Rate, Ratio, and Percent. These two units provide ample opportunity for calculator usage.

The Sample

Sixteen grade 8 students, eight from each of two Edmonton Public Schools, were selected for the project. Each student was provided with a Texas Instruments TI-30 calculator and given instruction in its use. Students were allowed to carry the calculator with them and were encouraged to use it in any way they wished -- in the mathematics classroom, in school in general, and outside of the school (at home, for instance).

Eight students were selected from one classroom (Teacher A) in one junior high school while six were selected from another classroom (Teacher B) in a second junior high school. Two students in the second school came from other rooms to join the six to form the Calculator Group. The two teachers, who provided some of the data for this project, were selected through the assistance of a mathematics consultant in one area of the city of Edmonton. The researcher requested that students be representative of a typical classroom -- males and females, students of high as well as students of low academic ability or mathematical achievement. As can be seen from the list of students, Appendix F, this was achieved by the teachers. They selected the calculator students through a process of student volunteer and teacher request and the researcher was satisfied that the students were a representative sample of grade 8 mathematics students.

From the school records a Lorge Thorndike Intelligence Quotient and the grade 7 final arithmetic grade was obtained for each student in the Calculator Group. These measures were used to identify students as high-achievers, average, or low-achievers.

The balance of the students from the two classrooms from which the calculator groups were formed, constituted the non-calculator groups. These were tested on certain dimensions for comparative purposes as indicated in the results of the research questions.

The parents of the calculator students constituted the parent sample for purposes of obtaining parental views on the use of calculators by grade 8 students.

Calculator Instructional Pattern

For the duration of the project, twelve weeks, the students met with the researcher for three forty-minute periods per week. Because of the method of organization of school schedules, in School A this occurred in three separate periods in the week while in the other school this constituted the entire Wednesday afternoon.

During the meetings with the calculator groups students were given instruction on the use of the calculator. The outline of much of the basic instruction is given in Appendix B. In addition, Appendix C contains worksheets which give a further indication of the type of work the calculator students were required to do. Appendix D provides a brief summary of the activities of each class period.

Instruments

Tests

The calculator students were given two in-class tests (as distinct from interview tests) in order to check on the ability of the students to write a key stroke sequence (KSS) and to do calculator computations.

Both calculator groups were given the Canadian Test of Basic Skills, Form 3M, both as a pretest and a posttest. This test provides norms for mathematical concepts and for problem solving. The full classes from which the calculator groups were selected were given researcher-constructed checkpoints as pre- and posttests on addition facts, subtraction facts, multiplication facts, division facts, and estimation. The two classes also were given researcher-constructed pre- and posttests, one on Rational Numbers and another on Rate, Ratio, and Percent.

All students from both classes were given an Opinion Check (an attitude assessment) both as a pre- and as a posttest in order to obtain some indication of change in attitude.

A second Final Estimation Check was also administered in order to get a better assessment of this skill based on a set of more difficult questions than those used on the original Estimation Check.

Each of the four operation checks was given a time limit of one minute. The task required the student to answer as many basic facts as possible in that time limit. The Estimation Check was given a generous time limit of five minutes in order to enable all but the lowest achievers to complete all twenty-five questions. The Final Estimation Check was restricted to five minutes in order to determine the total number of questions for which students could provide reasonable estimates. A "correct" estimate was so judged if it came approximately within ten percent of the correct answer.

At the end of the project parents of the calculator students were given an Opinion Check in order to determine their attitudes about the use of calculators by grade 8 students.

The project concluded after twelve weeks but the students were permitted to retain the calculators for an additional six weeks. A brief follow-up was then conducted. This took the form of a set of questions (Follow-up Checkpoint, Appendix E) for which students were required to provide the KSS and their answers.

The purpose of the questions was to determine whether students retained the ability to do specific kinds of calculations, some quite easy but others much more complex. The students were also asked questions about their use of the calculators during the previous six weeks.

Interviews

A total of six in-class personal interview tests called Checkpoints (see Appendix E) were administered during the project to determine the calculator ability of the students. The first took approximately thirty minutes per student, the next four about fifteen minutes while the last about fifty minutes (the longest took seventy minutes). The final one was lengthier in order to achieve a summative evaluation on calculator ability.

The two teachers participating in this study were interviewed informally from time to time. However, a more formal interview was conducted at the conclusion of the project.

Tape Recordings

During all class periods -- the class instruction including typical classroom discussions with question and answer periods, the group tests, and the personal interviews -- a tape recorder was used to record all conversation. The tapes were then transcribed and a written record of all verbal communications was available. It is from these transcriptions that much of the data were obtained in order to answer the research questions.

Observations

The researcher visited the classes a total of twenty-two times in order to observe the calculator students in their natural school setting. School A was visited ten times while School B was visited twelve times. All visits were made during regular mathematics periods. During the visits the researcher made notes of his observations. Specifically, observations were recorded about who used calculators, to what extent, and in what way were they used. The teacher's reactions or references to calculators or to calculator students were also recorded. The researcher assumed not only the role of an observer but occasionally that of a participant. However, the participant role was assumed to further the main goal of determining the utilization of the calculators or to assist students in determining an efficient sequence to be used in effecting a solution to a computation. The record of the observations provided further data used in the responses to the research questions. Furthermore, the observations helped determine some further research questions as indicated in the section listing those questions.

Analysis Structure

A number of research questions have been proposed. In order to answer these questions, data are used as obtained from the various methods described above. The tape recordings of class lessons and personal interviews provided data for many of the questions. Interviews with the teachers and the Parent Opinion Check provided additional information for other questions. Some of the questions were answered using the results of the standardized and non-standardized tests given to the calculator and to the non-calculator groups.

Summary

A large number of means of data collecting were used. This was done assuming that a wide collection of data would be more likely to show variations, similarities, and changes over a period of time.

Appendix E contains copies of the various tests, checkpoints, parent opinion check, and questions used for the final formal teacher interview.

A pilot study lasting two weeks was conducted by the researcher in December 1977. Six students were selected from one classroom. The purpose of the pilot study was to try some curricular materials and procedures with grade 8 students similar to the ones who would be used in the research project. All students were provided with a TI-30 calculator, the type that was purchased for the project. A number of observations were made as a result of the pilot study. A list of these is in Appendix G.

The Flow Chart, Table I, provides a summary of all the activities during the total project.

TABLE I

SUMMARY OF CALCULATOR PROJECT

Pilot Study (December 1977)

↓

Pretesting (January)

↓

Week 1 (Jan - Feb)
Introductory Exercises--gave first four pages

↓

Week 2 (Feb)
Further work on calculator functions
Pages 5 and 6 of practice exercises
Checkpoint 1

↓

Week 3 (Feb)
Constants of the operations
Estimation activities, ratio problems, complex fractions, pages 7, 8, 9 of practice sheets
Further calculator functions, special functions of the TI-30 (the "=" sign)
Checkpoint 1 completed

↓

Week 4 (Feb)
Group test 1, complex fractions, estimation worksheet

↓

Week 5 (Feb - Mar)
Complex fractions--practice in KSS
"Powers" worksheet, "Tower of Hanoi" problem, Patterns worksheet 1
Checkpoint 2

↓

Week 6 (Mar)
Large numbers problems, scientific notation practice, patterns worksheets 2 and 3
Constant of multiplication (review)

↓

Week 7 (Mar)
Negative numbers on the calculator
Integers test--some done in class showing KSS
Mid-project student survey
Patterns worksheets 4 and 5
Beginning work on fractions

↓

--continued

TABLE I (Continued)

Week 8 (Mar)

Use of x^2 and y^x keys--review
"Talking Calculator" worksheet
Three special problems--KSS required
Checkpoint 3
Relationship of expressions such as A ÷ B X C and A X C ÷ B

↓

Week 9 (Apr)

Problems I worksheet
Group test 2, percent problems, relationship of decimals, fractions, percent
Complex fractions
Fractions worksheet 1
Canadian football field and metric measurements
Checkpoint 4

↓

Week 10 (Apr)

Remaining "Patterns" worksheets--worked many examples in class
How to get exact answers beyond the 8-digit capacity of the calculator
Work with fractions--conversions
Worked Problems I worksheet questions in class

↓

Week 11 (Apr)

Checkpoint 5
Problems I--completed in class
Continued work on fractions--factoring in reducing fractions
Worked several problems from fractions worksheet
Estimation Exercises

↓

Week 12 (Apr)

Posttests (CTBS) (calculator groups only)
Repetands of more than 8 digits
Palindromes
Problems from fractions worksheets
Puzzle problems

↓

Posttests to total classes (Apr)

↓

Follow-up exercises (June)

CHAPTER IV

ANSWERS TO THE RESEARCH QUESTIONS

In this chapter each of the research questions stated in Chapter III is answered in light of the observations made. In stating excerpts from transcriptions sequential numbers are used to identify the excerpt and the number is followed by the student's code number; for example, Excerpt 1 (M-118) identifies the first excerpt and is attributed to the student coded as M-118. A missing code means that the excerpt is not attributable to a specific student or comes from some other source which the context should make clear. The conversation in each excerpt uses R to refer to speech by the researcher and S to denote the student. Where more than one student participated in an exchange the different students are denoted by the appropriate student code number.

Research Questions Related to Teachers

Q1--How do teachers adapt to the presence of calculators in the hands of some of their students in a grade 8 mathematics class?

Teacher A and the class were observed in ten different mathematics periods while Teacher B and the class were observed a total of twelve class periods.

During the twelve weeks duration of the project both classes worked only on the unit on Rational Numbers. At the beginning of the project both classes were working on operations with integers; this constitutes the first part of the unit on Rational Numbers. Before the project ended both classes had completed a study of traditional algorithms for operations with rational numbers although the whole unit on rational numbers had not been completed. Teacher A was just a few days from completing the unit while Teacher B had several sections to complete on solving equations and inequalities using rational numbers.

Each teacher was provided with a calculator (including an instruction book) identical to the ones supplied to the students. In conversations with the teachers early in the project, the writer referred to the calculators and how certain operations must be carried out. For instance mention was made that this calculator is programmed to follow the correct order of operations ($\underline{17 + 12 \text{ X } 13 =}$ would be done as 17 + (12 X 13) by the calculator). (Note: A single line under a sequence of calculations denotes a key stroke sequence, KSS). However no further attempts were made to force the matter of teacher interest or involvement with the calculators. With Teacher A there was no indication that the calculator was used or that the teacher became knowledgeable in its use. Teacher B became more involved. The calculator was on this teacher's desk (or readily available in the desk) on every occasion that the researcher visited the classroom. This teacher also had a desk top calculator available to the students at all times in the classroom. Students often borrowed this teacher's calculator if the desk calculator was being used; on occasion he loaned his calculator to students in the calculator group when such students neglected to bring their own. In the first two weeks of the project, when calculators were new to the calculator group and to the teacher, students were observed asking the teacher about the keys and the operation of the calculator. In these circumstances Teacher B would answer questions or try to determine answers by experimenting with his calculator or perhaps consulting the manual to help clarify a point.

During classroom observations in addition to the calculator students who usually had their calculators, some other students had calculators as well. Both teachers used the general policy that students could use the calculator for mathematics except that it would not be permitted for use during examinations. However Teacher B at the beginning of the project was reluctant to allow use of the calculator for all the mathematics classroom work except that students could do their assignment sheets and use the calculator to verify computations. Early in the project (by the third week) Teacher B modified his stand to permit full use of the calculator after each student had demonstrated knowledge of each new procedure or algorithm being studied in class. Shortly after this (by the end of the fourth week) this teacher further modified his stand to permit full use of the calculators by all students at all times during class mathematics periods but not for examinations.

It is pertinent to note that the Edmonton Public School Board policy for calculator usage in the junior high schools is simply that use is at the discretion of the teacher except that calculators are not permitted during examinations.

From the observations it is noted that the reaction of the teachers to calculators was to accept their presence but to make no other adjustments. Teacher B, in whose room was the desk top calculator, did have questions directed to him in the first few weeks of the project. These questions usually referred to the operation of the calculator; that is, to the proper sequence of feeding in a problem. However, neither teacher made any adjustments in his or her lesson presentations, in homework assignments, in sample problems, in work to be done at the chalkboard, or in any other aspect of his or her lessons, which could be interpreted as an adjustment due to the presence of calculators in the hands of some students. The calculators were treated simply as devices available to some students, who were allowed to use them or not as they pleased, during the regular mathematics lessons. Not once in twenty-two observations did teachers, as an example of an adjustment, give modified problems to be done by calculator students. Neither was there ever a separate discussion about possible algorithms that might apply for those attempting solutions using calculators.

Other than responding to questions on the specific operation of calculators, only three observations were noted where one teacher made a suggestion on format to students using calculators. Two observations occurred in one lesson in the seventh week of the project. The class was working on solving equations, an example of which is $\frac{1}{2}a + \frac{-3}{4} = \frac{5}{8}$. The teacher explanation dealt with the use of additive inverses and the class was told to write down all intermediate steps. The special instructions to the calculator group were that students using calculators should write down their calculations on the side of the page so that the work could be checked if an error was made. The second observation was made during the same lesson. During the assignment part of the lesson the teacher was assisting a student and noted that students with calculators that have a reciprocal key could use it to obtain reciprocals required. (Actually the explanation is correct only for reciprocals of numbers in decimal form since the key referred to is designated as $\frac{1}{x}$ which means that it gives $\frac{1}{x}$ where x is the number in display. The reciprocal of $\frac{12}{5}$, for instance, could not be found directly although $\underline{5 \div 12 =}$ would give the reciprocal in decimal form as would $\underline{12 \div 5 = \frac{1}{x}}$.) The third reference to the calculator groups was made in the eighth week by the same teacher. The reference was to the division of $\frac{20}{0.75}$. The teacher stated that this was difficult and somewhat awkward unless the students had a calculator but if they didn't then the teacher pointed out that it was probably easiest to do it as $\frac{2000}{75}$.

It must be noted that in the participant-observer mode of this portion of the project, it was decided by the researcher that he would not intervene in the teaching. The purpose was to observe what would happen under these circumstances but not to alter the situation through intervention. The teachers were made aware, through personal discussions with the researcher, that the purpose of his classroom visits was to see how the calculators were being utilized by the students and to see whether any difficulties were encountered in using calculators with specific problems. One teacher was given an indication in one conversation (see below) that alternatives provided by the teacher to accommodate calculator users would be appropriate. The matter of teacher reaction, adjustment, or any alteration in teaching was left for determination by the teachers with no direct urging by the writer.

In the sixth week, the half-way mark, Teacher B asked the researcher in general about the progress of the project and inquired, "Is there anything I could do in class or whatever?" This was the first direct question related to the calculator project. Previous references by the teachers were of a general nature inquiring how "things were going." It was during this conversation that the teacher was informed of the kinds of observations being made in class. Also the researcher added, "Any response you feel is appropriate in terms of the use of the calculator or in terms of doing anything in class that you feel would be appropriate for calculator users as different for the other students might also be the thing you could try." The teacher left the impression that he would likely try some things but this was not supported in future classroom observations.

In summary, the adaptation of these teachers to the presence of calculators in the hands of some students involved accepting their presence, occasionally assisting students in the operation of the calculators, and very minimally (three references in twenty-two observations) recognizing the probability of a different algorithm to be used when doing a computation using the calculator.

Q2--To what extent do teachers become knowledgeable about the possible role of calculators in a grade 8 mathematics classroom?

There is no doubt that the two teachers have definite ideas about the possible use of calculators in grade 8 mathematics. From the observations, discussions, and final Teacher Interviews, it is not clear whether these opinions were formed earlier and did not change or whether their association with this project did help to determine such opinions. It is probable that since the teachers did not choose to become fully immersed in the calculator project their opinions were likely formed prior to the project and likely modified only to a slight extent because of the presence of calculators. Nevertheless, some opinions were documented about the possible role of calculators.

One possible role identified was that the calculator could be used as a check on previously calculated quantities.

> Teacher B: I see the use of calculators at any grade level as an aid in checking their work.

There was an expression of concern about the ability of students to calculate or to "do arithmetic."

> Teacher A: I think as long as they're [meaning calculators] used properly it's fine. I'm not sure that in the unit you used the hand calculators [Rational Numbers] that's a particularly good place for them to be used because there they're essentially learning arithmetic, ..., and if they're learning it, I don't think they should be using calculators.

> Teacher B: There may be a fear that some students will not know or will not learn how to calculate if they use calculators.

Both teachers referred to a way in which they use calculators in grade 9 classes. From their comments and the use of calculators in this way, it is apparent that this is one role they accept for calculators.

> Teacher A: ... in grade 9 math they have a large geometry section where they have to calculate hundreds and hundreds of problems on surface areas and volumes and perimeters of plane figures. Now that, I think, is a really good area where calculators can be used because the student can perfectly understand, say, how to get the formula for the volume or something and perfectly understand how to calculate the answer and yet because they're careless they don't get it correct, ..., and for a lot of kids that are really weak in calculations, they almost completely give up because they know what's going on but can never ever produce a correct answer.

Apparently Teacher A accepts the fact that a student must be able to show, for example, that he knows that the surface area of a cylinder is $A = \pi DH$, substitute the values for π, D, and H and then be allowed to multiply these on the calculator. Teacher A does concede that such students should not be judged on their inability to correctly find the product by pencil and paper computation. Yet this teacher did say earlier that calculators should not be used when students are "essentially learning arithmetic." It appears that Teacher A would distinguish between units or portions of units in which computational skills are required and those in which computation is required for solving a problem. Teacher A referred to the fact that a calculator should not be given to grade 7 students since essentially it's an arithmetic program. This teacher also went on:

> Teacher A: Students even in grade 9 don't know how to do arithmetic consistently and well. Adults don't know how to do arithmetic consistently and well so what's the point of giving them a crutch in a section where they're supposed to be learning that skill.

Teacher B also allows grade 9 students to use a calculator quite freely. Teacher B mentioned that the class referred to is "a top-notch class." The observation made was that even such grade 9 students use the calculator extensively but at times too readily. Teacher B draws attention of the students to such use whenever he notices it in order to keep students from using the calculator when a computation is too easy. The case cited by Teacher B to illustrate the point is when students reach for the calculator to do 15 X 12. Teacher B reminds students that 15 X 12 is 30 X 6 and hence not a computation to be done on a calculator.

Teacher B also feels that different students have different alternatives. Some students, it was observed by the researcher, reach for the calculator to multiply a number by 100 or to take 10 percent of a number. Teacher B claims that slow students have only one route, the calculator, and they use it. Better students will have two routes claims Teacher B, a calculator route or a mental calculation route. They would automatically choose the easier one, in this case a mental computation.

Q3--What is the attitude of the teachers about the possible use of calculators in grade 8 mathematics?

Some of the examples given in answer to Q2 would apply here. It is obvious that Teacher A feels that students in grade 8 should know how to do the computations using pencil and paper. Having proved this ability it is then quite permissible to use the calculator. On the other hand Teacher A concedes that if your objective is to do mensuration questions the full use of the calculator should be permitted. Teacher A relates this to the different objectives for these lessons.

Both teachers referred to School Board policy and accepted these restrictions. However they both felt that some more definite policy is needed. Teacher A felt that the policy should set out the kind of activities for which the calculator could be used. Teacher B stated that likely school programs will be adapted to the calculator.

> Teacher B: It [the curriculum] has to change. Probably even what you're doing now is going to give us a lead as to what direction this is going to take.

Teacher A favors the use of calculators in a way which will develop certain skills. This teacher referred to an activity used by another teacher, in which students did rounding off and then checked their accuracy by using the calculator. This teacher favors any activities in which the calculator is used to enhance any such skills such as estimation skills or when the calculator enters into a task, "... sort of for interest, for motivation, also to get the arithmetic out of the way." The last comment concerning arithmetic referred to using a calculator to practice rounding off skills.

Teacher A would certainly not favor much reduction in computational practice exercises to be done with pencil and paper. When the possibility of a core computation requirement was mentioned with the suggestion that more complicated calculations (such as a three-digit number multiplied by a three-digit number) be left to the calculator, this teacher disagreed.

> Teacher A: A student in junior high isn't going to get facility in multiplication unless they do get hours and hours of practice, and if you teach them and show them the algorithm and suppose you do take it to a three-digit times a three-digit number and you give them one or two questions, they aren't going to remember how to do it and they're not going to be able to do it.
> R: Do you think it's important that we be able to do such multiplication?
> Teacher A: Yes. Someone who can calculate well can get answers to, say, multiplication questions faster and more accurately than someone on a calculator who doesn't know what they're doing.

This teacher elaborated on this theme by stating that some students start calculating and get confused. Another student will think about a question, do the calculation by pencil and paper and be done long before the student with the calculator. This teacher obviously feels strongly about being able to do all these procedures without the calculator before being permitted to use the calculator freely.

> Teacher A: But yet the students should be able to do the calculations without the calculator and that is the thing that is really important.

Research Questions Related to Regular Student Learning

Q4--To what extent do grade 8 mathematics students learn to use a calculator effectively to do basic mathematics calculations?

In the fourth week of the sixteen-week project students were given Group Test Number 1 (Appendix E). The following nine questions could be described as requiring straight-forward calculations:

2. 16 + (89 X 3) = , 3. 387 - (466 + 466 + 243) = ,

4. 783.47 - 69.874 = ,

7. 948 + 948 + 948 + 948 + 948 + 948 + 948 + 948 = ,

8. 8473 X 0 = , 9. 1 X 7439 = ,

11. 0.010 816 ÷ 0.001 04 = , 12. 888 X 888 ÷ 4.4 = ,

13. 7946 X (27.93 ÷ 0.3) = . In addition number 5. 7483 + [ ] = 4096 required only moderate interpretation and becomes a basic calculation. Number 6. 643 + 929 + [ ] = 2617 and number 10. The product of 768 and 237 = , require some further understandings before applying direct calculation.

Number 2:

The question is 16 + (89 X 3) = and the most direct answer would be 16 + 89 X 3 = . All students did this question correctly. Of sixteen students five used a left parenthesis, 16 + (89 X 3 = , which is not needed because the calculator used in the project is programmed to observe correct order of operations; four students used both parentheses, which is a further unnecessary keystroke; two students reversed the order, 89 X 3 + 16 = , which is unnecessary but is the sort of thing one would do with a calculator that does not do correct order of operations.

Number 3:

This is 387 - (466 + 243) = and the most direct way to obtain the answer would be 387 - (466 + 243 = . Eight students did it that way, four used both parentheses while four did (466 + 243) - 387 = and so obtained the additive inverse of the correct answer, 322 instead of $^{-}322$.

Number 4:

This question, 783.47 - 69.874 = would be done exactly as shown. All sixteen students did exactly that and got the correct answer.

Number 7:

This question should have been done as 948 X 8 = in view of classroom exercises (even if they didn't know this from other experiences). Nine students did that, one did 948 X 7 = so miscounted the number of addends, three correctly used a constant of addition (948 + K = = = = = = = =), two did it correctly by adding in sequence, and one used 948 y^x 8 = . The latter misinterpreted it as a multiplicative situation rather than an additive one.

Numbers 8 and 9 were intended to see if students would do obviously simple items by mental calculation. Only seven did number 8 mentally, nine did it with the calculator, but all got it right. Number 9 was done mentally by four students, twelve used the calculator, and all got it right.

Number 11:

The most direct solution for this one is 0.010 816 ÷ 0.00104 = except that in entering these numbers the '0' preceding the decimal is omitted. Fifteen students got this correct while one indicated correct KSS but neglected to indicate the solution.

Number 12:

This one, 888 X 888 ÷ 4.4 = , could be done most efficiently as 888 x^2 ÷ 4.4 = but at this stage students may not have had many opportunities to use the x^2 key. However five students did use it, nine did it directly as given getting the correct answer, one did it directly but enclosed 4.4 in parentheses, while one inadvertently omitted the question.

Number 13:

This is 7946 X (27.93 ÷ 0.3) = and the most direct route would be to do it as 7946 X 27.93 ÷ 0.3 = since parentheses are not needed. Seven students indicated they did it using only the left parenthesis but one got an incorrect answer, eight used both parentheses while one reversed the order doing the division first followed by the multiplication (he had the correct answer).

Number 5:

This question, 7483 + [ ] = 4096 would require students to know that subtraction is needed so

4096 - 7483 = would be the easiest way to get the solution, $^{-}$3387. Thirteen students got this correct, two reversed the order and got 3387 while one omitted this question (and the next one).

Number 6:

This was similar to number 5; 643 + 929 + [ ] = 2617 except that students would need to subtract two numbers so the solution would be 2617 - 643 - 929 = to get 1045. None used the direct method but six did it by using a left parenthesis, 2617 - (643 + 929 = while four others did it similarly but used both parentheses. Three students correctly used memory by doing 643 + 929 = STO 2617 - RCL = which is correct but not as efficient. One student added 643 and 929, wrote the result and then subtracted for a correct solution. One started with 643 + 929 but the remaining KSS is not clear but he got the correct answer. One student omitted this question (she had also omitted number 5).

Number 10:

The students needed to know that 'product' calls for the result of multiplication. Nine did this correctly while seven correctly found the sum of the two numbers so the terminology was the problem apparently.

In summary if the criterion is to use the calculator to get the correct answer, then of the 144 correct answers possible on the first nine questions, 134 or about 93% were correct. However not all of these were efficient. Of the last three requiring some further interpretation the correct solution was obtained in 37 of 48 possible ones or about 77%. The problem with poorest results was number 10 which called for the product of two natural numbers; the problem for seven of sixteen students was the meaning of "product." If efficiency is a criterion then the percentage is 74 for the first nine problems; using both parentheses where one will do was interpreted as efficient as was reversal of order but obtaining the correct solution. Inefficiency was assumed if the calculator was used to multiply by 0 or by 1. Under these criteria (and assuming use of memory in number 6 as efficient), about 73% were efficient in the last three problems (5, 6, and 10).

In view of this evidence students generally do not have much difficulty in doing basic calculations although they are not always most efficient in performing the calculations. The fact remains, though, that getting the correct answer is done with a high degree of accuracy (93% overall). When some interpretation is needed or when negative integers enter into the calculation this work does remain trivial for many students.

Q5--Will the use of calculators improve the students' ability to estimate answers to questions involving basic operations?

A total of fifty students did the Estimation Check of twenty-five questions (Appendix E). Five minutes was allowed but this was set so that all but the very slowest would get through the check. Thirty-four non-calculator students went from a pretest mean of 10.2353 to 13.2941, a gain of about 3.05 points, while the sixteen calculator students went from 12.3125 to 16.6875, a gain of about 4.38 points.

A t-test using 48 degrees of freedom shows that at the 5% level there is no significant difference between the mean of the samples on the pretest. Under the same conditions there is no significant difference between the means on the posttest.

Students were also given a Final Estimation Check (Appendix E). This test, made up of fifty questions considered by the researcher to be more difficult than the Estimation Check mentioned above, was given to all students at the conclusion of the project. Students were given five minutes for this test and only two students (of fifty) attempted all problems. The mean of the calculator students was 13.1875 and of the non-calculator students 9.3529. The difference between the means was significant at the 5% level. The calculator students made 12.5 mean number of errors while the non-calculator students made 15.8824.

In summary, it is not conclusive that the use of the calculator helped to improve the ability of the students in their estimation skill. On one Estimation Check the calculator students scored significantly higher than the non-calculator students, but it is not possible to attribute this to the use of the calculators exclusively. It may be that in general the estimation type of activities given throughout the calculator project were the important determiners. Nevertheless, the calculator students did improve in estimation skills and in one case did significantly better than the non-calculator students.

Q6--Can grade 8 students effectively use calculators to assist with calculations required for traditional algorithms associated with operations with rational numbers?

The results of two questions will be analyzed. Questions 5 and 6 of Checkpoint Number 6 (Appendix E) asked students to find $16\frac{3}{7} + 38\frac{14}{19}$ and $27\frac{4}{7} \times 36\frac{14}{15}$ and to express the answer in the usual common fraction form. Since these questions were given at the end of the project the results show the capability of the students after considerable experience with the calculator.

In doing $16\frac{3}{7} + 38\frac{14}{19}$ the normal route would be to use the calculator to find 7 X 19, 3 X 19, 7 X 14, and record $16\frac{57}{133} + 38\frac{98}{133}$. Then using the calculator the student would likely get $54\frac{155}{133}$, from which he would get $55\frac{22}{133}$. By using the calculator to check divisibility the student could quickly establish that $\frac{22}{133}$ is in lowest terms.

Except for two students, M-117 and F-120, the students were able to work this question readily, to the point where they got the final correct result, $55\frac{22}{133}$. Student M-117 had trouble getting from $54\frac{155}{133}$ to $55\frac{22}{133}$. Constant step by step help was needed to get that conversion. Student F-120 needed constant assistance from the very beginning:

Excerpt 1 (F-120):

[After the researcher had reviewed what the question called for.]

R: What should we do?

S: We should change the fractions [meaning to improper fractions.]

R: It's probably better in addition to work with them as they are but change only the fraction portion. [Researcher wrote 16—— + 38—— = to assist in the process].

How do we change the two fractions? [No reply.] When you want to add $\frac{3}{7}$ and $\frac{14}{19}$ what must you do with them?

S: You find the common denominator?

R: Right. What is a common denominator that you can *always* use? [No reply.] You can always find one by multiplying the denominators, in this case 7 times 19. Do that and enter it. [Student now had $16\frac{\quad}{133}$ + $38\frac{\quad}{133}$ = .] Now we must change $\frac{3}{7}$ to that one. [Showing $\frac{\quad}{133}$.] What do you do?

S: 19 times 3. [Student did that and for the next one did 14 X 19 and was corrected by the researcher.]

[The questioning continued in this way with the researcher guiding progress with questions as follows:]

R: Now we're going to add so what is the sum of the whole numbers?

S: 16 + 38. [Student did this on the calculator so now had $54\frac{155}{133}$.]

The next stage was the change to $55\frac{22}{133}$ which presented a problem (a later discussion illustrates this).

Most students (nine of sixteen) proceeded directly to work on the question getting to $55\frac{22}{133}$ with no difficulty. One student, F-112, whose calculator technique was the most careless of all students, made an error in calculation which the researcher corrected. Six students, M-117, M-123, M-121, F-119, F-112 (mentioned above), and F-120 (mentioned above), wanted to begin by changing the mixed numerals to improper fractions. The researcher suggested working with the mixed numbers as indicated in Excerpt 1 above.

The biggest difficulty experienced in this problem was in the decision concerning whether $\frac{22}{133}$ can be reduced. Clearly this is not a calculator problem but one of mathematics. Generally all students, even the ones weakest in mathematics, know that some process of division was necessary to check this out. In all cases students used the calculator to decide whether 133 could be divided by 11.

Following is an excerpt which is typical of students who needed some guidance but would likely have answered the question if left completely alone:

Excerpt 2 (F-126):

[Student had $55\frac{22}{133}$.]

R: How do you know if you're finished?

[Student did 133 ÷ 3 = .]

Why are you dividing 133 by 3?

S: Because it can't be divided by 2.

R: Yes, but what are the numbers that divide 22?

S: 11 and 2

R: So what can you do to check if 133 can be reduced?

[Student did 133 ÷ 11 = and noted that it doesn't divide.]

So what do you now know?

S: That's as low as it can go [meaning the fraction can't be reduced].

Appendix J contains excerpts from work with students M-117 and F-120. These students had by far the most difficulty after they got to $54\frac{155}{133}$. These two were the ones mentioned above as needing extra assistance.

Question number 6 required that the students do $27\frac{4}{7}$ X $36\frac{14}{15}$ and get the answer in the same mixed number form. Essentially this meant expressing the question as $\frac{193}{7}$ X $\frac{554}{15}$ and then multiplying to get $\frac{106\ 922}{105}$. By division this gives 1018.3048 which indicates that the answer is 1018 plus some fraction. The procedure of 1018 X 105 = EXC - RCL = (provided 106 922 had been put into memory earlier) yields 32 which now indicates that the answer is $1018\frac{32}{105}$. By inspection one observes that $\frac{32}{105}$ is irreducible although that is not obvious to a grade 8 student.

In this question students had no difficulty in doing the proper calculations and then writing $\frac{106\ 922}{105}$. Seven students had to be reminded that in this multiplication question it is easier to work with each fraction expressed as an improper fraction. Perhaps as a result of just having done an addition one previously, they wanted to attempt working with the numbers in mixed number form.

Students had difficulties in this question in using the calculator to get from $\frac{106\ 922}{105}$ to $1018\frac{32}{105}$. The division yields 1018.3048. Three students wanted to multiply 0.3048 by 105 to find the fractional portion. The researcher pointed out that this may not always lead to a whole number (it does not in this case) and one could not be certain that the correct value has been found. In the case of twelve students the researcher had to assist them usually by pointing out that it is necessary to multiply 1018 by 105 and then subtract that result from 106 922. To get this subtraction the students had been reminded to put 106 922 into memory (STO key) when that number was first obtained. When 1018 X 105 was done the value 106 890 was obtained and in display. At this point EXC - RCL = would put 106 890 into memory, 106 922 into display and then subtract in the correct order. Of the twelve students needing assistance in doing this portion, eight needed help with the correct KSS for exchanging the display and memory and completing the subtraction. One of the students, F-120 referred to previously, could not do this question. She completed it only with assistance throughout with much of the information being given to her directly by the researcher.

It is noted that the EXC function of the calculator exchanges the display with the value in storage. The RCL key simply recalls from storage without destroying the value in storage, the STO key is used to place the displayed value into storage and SUM adds the display value to the storage value and retains the sum in storage. Appendix A, Definitions, contains more on the TI-30 calculator. More on the ability of students to use these keys is contained in the reply to Q16.

The next excerpt is used to illustrate the kinds of exchanges that occurred in assisting students.

Excerpt 3 (M-125):

[Student got to $\frac{106\ 922}{105}$ on his paper and the numerator had been placed into memory upon the researcher's request.]

R: Now what do we do?

S: Reduce it. [He did RCL ÷ 105 = and got 1018.3048.]

R: Now we have to calculate how much of 106 922 has been used to get 1018 and so find out how much of it is left. [Student did not know so the researcher told him to multiply 1018 by 105. Student began to enter all of 1018.3048 and the researcher had to tell him to use only 1018. The student did and got 106 890 and was told that subtraction is required.]

R: Where is 106 922?

S: In storage.

R: Right. How do we get 106 922 minus 106 890?

S: Recall.

R: Well, if you do, you will lose the number in display.

S: Well, exchange them.

R: Right. [Student did EXC - RCL = , got 32 and wrote $1018\frac{32}{105}$.]

The final difficulty that thirteen students had was in deciding whether $\frac{32}{105}$ could be reduced to lower terms. Three had no trouble here. Three of the ten needed only minor guidance while seven needed considerable help. The next two excerpts illustrate respectively a case of slight assistance and one with a large amount of help. The latter continues Excerpt 3.

Excerpt 4 (M-115):

R: Can you reduce $\frac{32}{105}$?

S: 2, 3 won't work.

R: What can divide 32?

S: 2

R: And any other number that doesn't involve 2?

S: [Pause] No, because they'd all be multiples of 2.

R: Yes, so?

S: That's it; that's as far as you can go.

Excerpt 5 (M-125):

[A continuation of the situation described in Excerpt 3.]

R: What's one number you can divide into 32?

S: Well ....that's .....

R: What's the simplest number we can use?

S: Well, 1 can go.

R: Yes, but other than 1.

S: I think 2.

R: Yes. Will 2 divide 105?

S: No.

R: Fine. I can't divide 32 by anything except 2 and then 2 again but that's all, but 2 doesn't divide 105 so what can we say about the fraction?

S: It stays the same.

Only one student, M-115, did this question without assistance from beginning to end.

From the above analysis it is clear that students can use the calculator to assist them with certain steps to do rational number algorithms. However if left entirely to themselves it is probable that no more than eight of sixteen (50%) would likely have been able to work through to the final solution. Students need much more practice to develop skill in the algorithms necessary. Steps involving use of storage were particularly difficult. In the time available not all students became aware of the power of this aspect of the calculator. An algorithm which was difficult to master was EXC - RCL = , used to get A - B when A is in storage and B is in the display. Since in the project, the work with fractions came near the end, apparently not enough time was available to make these procedures clear to all students. Nevertheless, with the more direct calculation parts of these problems students had no difficulties. That is, in getting from $16\frac{3}{7} + 38\frac{14}{19}$ to $55\frac{22}{133}$ and from $27\frac{4}{7} \times 36\frac{14}{15}$ to $\frac{106\ 922}{105}$ most students had no difficulties. In the first the major difficulty was in deciding whether $\frac{22}{133}$ was in lowest terms. In the second the major problem was in going from $\frac{106\ 922}{105}$ to $1018\frac{32}{105}$ and then again the difficulty with the concept of relatively prime numbers arose.

It should be noted that since no accommodation was made in the schools for calculator activities, these students did not have any practice on the calculator algorithms. In addition the kinds of fractions given in the classroom would not give students opportunities to practice factoring of large numbers and then using these in reducing fractions. The only chance students had to do these was in the class sessions with the researcher and in practice exercises done on their own. However, these would not be sufficient to firmly entrench these new procedures.

Q7--Will the calculator enable grade 8 students to cope with larger numbers (for example $12\frac{13}{17}$ as compared to 8½) in doing rational number algorithms?

There is no evidence to indicate that more unusual fractions, such as $38\frac{14}{19}$ used in Q6, present any more difficulty to students than would simpler fractions. In working with fractions students used the algorithm 38 X 19 + 14 = and then wrote $\frac{736}{19}$ if they needed to work with fractions. In doing questions 5 and 6 from Checkpoint Number 6, students made no errors (except in one case) in actually obtaining the equivalent fraction forms. As shown in Q6 the difficulties were with such things as knowing whether they should work with mixed numbers or improper fractions, how to go from an improper fraction to a mixed number, and how to decide whether a fraction is in its lowest terms.

During class instruction students worked through several problems using difficult fractions. However, the better calculator users are usually the ones who provide most of the answers as to procedures. The mathematical principles also are supplied by those who are high-achievers in mathematics. It is obvious from the difficulties seen in the personal interviews that the average or below average students (in general ability as indicated by the IQ) have more difficulty in working through examples and need much more guidance. See Appendix J (Excerpts 3, 4, and 5) for examples of in-class discussions while working on rational number operations. In all of the excerpts in Appendix J, it can be seen that the magnitude of the mixed numbers had no bearing on the ability of students to handle the calculations. It was relatively easy for many (but not all) students to factor 14 117 to get 17 X 743 and then to decide that neither of these factors divides 34 817. The difficulties for students, except for the few high-achievers, was to know that it is a factorization that is needed and subsequently a trial of the factors for divisibility into the denominator.

An inspection of the grade 8 mathematics program used in these schools soon demonstrates that the calculation practice exercises, as well as calculations required in problem solving, are meant to test the students' ability to apply an algorithm and not to work with more 'unusual' number forms. Consequently questions such as $2\frac{2}{3} - \frac{1}{6}$ or $^-2\frac{3}{4} - 3\frac{5}{8}$ or $2\frac{1}{2} \times \frac{5}{11}$ are common. In fact these are among the more difficult ones when you compare to such as $\frac{1}{2} \div \frac{1}{3}$ or $3 - 1\frac{1}{4}$. Students cover literally dozens of such exercises in order to establish the rules of the operations. Opportunities for any further aspects of work with rational numbers are rare if existent at all. It is doubtful whether students experience or appreciate the algebraic nature of the algorithms when they are never required to do any but the simplest kinds of fractions. This is a question that bears further investigation.

Q8--Will students prefer working with rational numbers in decimal form for various operations rather than in fraction form?

In doing problems in class instruction conducted by the researcher, instruction and practice were given in doing rational number algorithms in both forms. In a question such as $86\frac{7}{12} - 38\frac{14}{15}$, $47\frac{13}{20}$ is obtained and the calculator is used in the various calculations. In the decimal form approach a relatively simple algorithm of $\underline{86 + 7 \div 12 = - (38 + 14 \div 15 =}$ would yield 47.65.

In Checkpoint Number 6, students were required to do two questions, $16\frac{3}{7} - 38\frac{14}{19} =$ and $27\frac{4}{7} \times 36\frac{14}{15} =$ (numbers 5 and 6 analyzed above in Q6) giving the final result in fraction form. In addition the students were asked to do two questions (numbers 7 and 8), $18\frac{7}{9} - 12\frac{10}{11} =$ and $33\frac{7}{12} \div 4\frac{15}{22} =$, giving the answer in decimal form. The latter implies that the whole question could be done in decimal form in order to obtain a final result in that form.

Upon completion of the checkpoint students were asked some questions about their reactions to the calculator project. Among the questions was one asking them about their preference in working with rational numbers. Fourteen students enthusiastically chose the decimal form of doing these problems. The most common reason given was that it is easier in that form. Two students (both above average) also added that in decimal form one doesn't need to write down so much. One of these students added, "... even though writing down stuff helps you." (He had contradicted himself because just prior to this he had been advocating pencil and paper work since this provided a written statement of the work). The other student added as an afterthought that he didn't really mind the fraction form as it wasn't all that hard with the calculator. Two students, M-117 and M-125, stated that they would prefer the fraction form. The former, a low-achieving student in calculator usage and in mathematics, preferred the fraction form, "Because it seems easier to me, because all I do is figure it out." The reference to "figuring it out" meant that he could do parts of it and write these down, whereas the decimal form requires a continuous flow in the KSS. This student was very deliberate and slow in using a calculator. Usually he verbalized his KSS in a low tone as he worked. He did concede that the decimal form took much less time. It is possible that the fraction form gives some students some security since it is a form with which they are more familiar. The decimal form is not part of their regular classroom algorithms.

The conclusion is that students do prefer the decimal form of working with rational numbers.

Research Questions Related to Student Learning Beyond Standard Curriculum

Q9--Because of their ability to use the calculator, will grade 8 students go beyond curriculum requirements in mathematics?

In the first week of the project two students, M-124 and M-122, asked about the meaning of trigonometric function keys on the calculator. By way of illustration the researcher posed a typical "find the height of a flagpole" problem by being able to measure the shadow cast by the flagpole and the angle of elevation of the top of the pole taken at the end of the shadow. This led to a brief description of the tangent function. After calculating the height of the flagpole no further questions were posed nor did students ever return to these functions with further inquiries. The one example seemed to have satisfied their inquiry about these keys on the calculator.

At both schools during the first week questions were raised about the "log" and "lnx" keys. The researcher's explanations were to the effect that these are used in advanced mathematics taken in high school and in university. A brief explanation of the meaning of the log of a number was made by referring to power notation. Again this was never pursued by any students either in class or as an extra during appropriate times.

The researcher noted several times that some students did much casual exploratory work with their calculators. However most of this was not related to discovery (or attempted investigation) of advanced mathematics. This exploration seemed to be directed to seeing what happens when various keys were pressed singly, in pairs, in groups, or in various sequences. One student, for instance, discovered that he could turn the calculator on by depressing certain groups of keys in a certain sequence but not using the ON/C key.

In summary, these sixteen students stayed with the standard curriculum. They did not attempt investigations into topics not normally covered in their grade. They were satisfied to do their work, either in the calculator periods or in their mathematics periods, using the calculator if it seemed appropriate or necessary.

Q10--What aspects of Rational Numbers and Rate, Ratio, and Percent will be trivial for students knowledgeable in the use of calculators?

The traditional work (especially as it appears in the consortium material and used in many schools of the province) is based on simple numbers. The work is usually such that the numbers are "nice." For instance

$8\frac{1}{3} \div 4\frac{1}{6}$ becomes $\frac{25}{3} \times \frac{6}{25}$ and everything works out to simple calculations. In the sense that these questions require the students to know the fundamental procedures, they are a sufficient challenge. However they do not provide much of a challenge for good calculator users. Using the calculator, students would do $\underline{8 + 1 \div 3 = \div}$ $\underline{(4 + 1 \div 6 =}$ and get 2. The invert-and-multiply rule is not of any significance when this occurs.

Similarly, when working with Rate, Ratio, and Percent problems, students are given simple questions such as finding 15% of 70. They do this as $\frac{15}{100} = \frac{N}{70}$. The calculations involved are simple because the emphasis seems to be on the method. To find 30.25% of 78.9 is really no different but such a problem is not likely to be raised in the classroom. When faced with $\frac{30.25}{100} = \frac{N}{78.9}$ the difficulty to the students without calculators is in the calculation. However, when students have calculators the latter question is no more difficult than the former.

The same techniques of teaching can go on but the actual calculations will be trivial. All student concentration can be on the method and on the analysis.

From the excerpts given previously and from those in Appendix J, it can be seen that the difficulty students have is usually with the method to be used, not with the calculations, when they have the use of a calculator. Their common problems are to decide what numbers must be multiplied or otherwise processed. Once that is known the actual calculations are usually simple. Only in the more complex situations presented do some make errors of entry or others related to calculator manipulations.

Q11--Because of ability with calculators will students pursue various topics not in the standard curriculum upon the suggestion of the researcher?

Several opportunities occurred when the researcher broached topics that might be new to grade 8 students and which would not be part of the standard curriculum. One such instance was when students in School B asked about the trigonometric keys on the calculator. The researcher taught the use of the tangent function (described in Q9 as the "flagpole" problem) to this class. He also introduced this in School A. As mentioned before the students showed keen interest and were able to follow the reasoning in the problem. There was no further interest expressed by students at either school nor did students ever ask about the use of these keys again. The researcher did not follow up on this type of problem on his initiative either.

There were other instances when the researcher initiated activity into areas not normally found in the grade 8 mathematics curriculum. One such case was in working with powers. Students were given several problems in the form of a worksheet (see Appendix C--Powers). In spite of much encouragement and assistance students could not work through all the problems. Nor did they complete these problems when a start had been made in class. The problems were completed by the researcher leading the class in discussion. The classes did not volunteer any information that could be interpreted as student initiated extension of a topic introduced by the researcher.

Another example of work that provided opportunities for students to continue explorations is illustrated by the worksheet called The "Talking" Calculator (Appendix C). All students worked at it very diligently and were most anxious to get the message without outside assistance. However, having done this, there were no further moves by the students to produce mathematical examples of their own which would produce words. They did inquire about all the letters that could be produced from the numerals and many students did use these numerals to produce words. Generally this was done in class during the time allotted to do this work on The "Talking" Calculator questions. They did not, however, return at some later date with additional follow-up activities related to this exercise.

Q12--Will students show interest in following up on mathematics problems posed by the researcher and develop their own algorithms in solving these problems?

The intent of this question was to see to what extent students would take the initiative in working special problems, in seeking other problems of a similar nature, or in developing alternative algorithms to solve certain computational problems.

A number of situations arose or were provided by the researcher for demonstrating interest in this direction. One such example is the worksheet, Problems I (Appendix C). This provided a number of word problem situations which could give rise to further explorations or to statements of similar problems. However no such moves were made by any students as a result of doing the Problems I worksheet. In fact most students had difficulties with these problems and consequently made only a beginning by attacking the first problem. Several students were incorrect even in this minimal attempt. The difficulties encountered repeatedly related to mathematical knowledge (for example, should you multiply or divide). Students were forever uncertain as to what to do in order to solve the problems. Once the procedures were clarified, the actual calculation was not a problem. As usual, it was the high-achieving students who were able to assist in deciding on the method to be used in solving problems.

Research Question Related to High-achieving Students

Q13--Will high-achieving students be much more active in exploring and going beyond basic requirements than will low-achieving students?

As might be expected good students provided most of the input when difficult (and unusual) problems were discussed in class. However as mentioned previously in Q12 there was not a great deal of activity showing that students explored mathematical problems or went into more depth in topics that were raised. Most of the exploration occurred in seeing how the calculator works, what effects certain keys produce, and such general exploring of the operation of the calculator. Again this seemed to be predominantly an activity of high-achieving students.

This question will be more thoroughly explored in the next chapter in Case Study Number One in which the calculator ability of a high-achiever is explored.

Research Question Related to Low-achieving Students

Q14--Will low-achieving students be able to do basic calculations efficiently using the calculator; will they limit their calculator activities to minimum curriculum requirements?

An inspection of the worksheets shows that students had a tremendous amount of opportunity to do various kinds of problems and calculations. In general, students (with the exception of three) did not do worksheets outside of the in-class opportunities. Therefore many students did not complete many of the worksheets, especially those that were more challenging. Some worksheets, such as Problems I, were completed through class-instructor discussions but those that were not completed in class, were often left undone. The low-achieving students invariably did not complete worksheets.

Among the worksheets were several questions involving some sequences for the calculation of π. The concept of π was discussed in class and some of the sequences evaluated to a specific point. Then students were left to try the others on their own. However, an inspection of the worksheets showed that students did not attempt these. In working these in class, only the high-achieving students could assist in class discussions to devise algorithms while the low-achieving students would attempt only basic calculations.

Low-achieving students can use the calculator efficiently only for basic calculations. They can easily do questions involving the basic operations as seen in the analysis for Q4. Chain calculations can readily be done by all students (recall that the TI-30 calculator does follow the algebraic order of operations). However, it is interesting to note that in the follow-up when students were given basic calculations, some forgot that the calculator is correct in this respect and inserted parentheses where none were necessary, or altered the question by inserting parentheses.

Research Questions Related to the General Use of the Calculator

Q15--What are the necessary techniques for effective calculator use?

It was observed that in the classroom most, but not all, students used the calculator by holding it in the left hand and operating the keys with the right, usually while holding pen or pencil in that hand. Only a few generally left the calculator on the desk and operated it in that way. The most common way for these students was to operate the calculator using the right hand. However a number of students used the left hand, a technique that had been suggested to them by the researcher early in the project. This seemed to be the more predominant technique in the personal interview. Although this was efficient in terms of hand movements and hence in terms of time, it was no more or less accurate than other methods.

One student, F-112, who was the most careless in terms of how the calculator was handled, almost always picked it up, rested it on the fingers of both hands and then manipulated the keys with the thumbs. During the in-class discussions this student tended to have incorrect results and often raised questions because of these. In addition this student, more often than anyone else, would recalculate because of the discovery of errors. It should be noted that her two-thumb technique may not be the cause of her errors of carelessness, since this technique has been mentioned as one possible way of handling a calculator effectively.

It has been suggested that the use of the non-writing hand would be an efficient way of operating the calculator. Furthermore, the operation should be similar to that of a typewriter -- certain fingers operating certain keys and the use of a touch system allowing the operator to observe the display or the working materials. Some calculators even have the '5' key constructed differently making it discernible to the touch (a raised dot on it or having a slight concave construction). From the experience of this project it does not appear appropriate to make such operations mandatory for grade 8 students. Furthermore, calculators are not standardized and hence mastery of one would not be directly transferable to another. However, encouraging the use of the non-writing hand for operating the calculator would be an advantage.

Because of the uncertainty surrounding calculator manipulation, further research is needed to determine the best technique.

The most efficient calculator users, those who got answers quickly and accurately, had developed in certain skills and techniques. However, not one student could be classified as being excellent in ability with the calculator. Yet all students were able to manipulate the keys successfully. The more proficient calculator users tended to check the display to see whether the number entered was the correct one. Whenever any doubt arose as to the accuracy of the result all students would recalculate for verification. It was typical of students to do all calculations on the calculator even though parts of a series of calculations could be done mentally very readily. For example in doing 16 X 7 + 3 students would invariably enter all of these steps on the calculator. Sometimes the better ones would remark that they could just add 3 but meanwhile they would actually calculate it.

Uncertainty about their own recollection of arithmetic rules led to reliance on the calculator. Faced with 8746 ÷ 100 the students resorted to the calculator since they were uncertain whether the decimal should be moved right or left. Having done it on the calculator they would feel reassured that the rule they thought of holds and would often remark that, "Oh, yes! You just move the decimal two places to the left." Only the very high-achieving students would give such an answer directly and confidently without feeling the need for verification on the calculator. If the question was 8746 ÷ 100 + 753 then the students would automatically enter the whole thing into the calculator. The series of operations apparently cued them to rely on the calculator. Even something like 9 X 8 + 39 would likely evoke 9 X 8 + 39 = , except again in the case of the high-achieving students, who would enter 72 + 39 = . If the question was 9 X 8 + 30 = then the high-achieving students would most probably give 102 directly without resorting to the calculator. The low-achieving students would calculate the whole thing.

The students did not tend to estimate results or tend to be alert to the reasonableness of answers. In Checkpoint Number 6 (Appendix E) number ten posed the situation of a student calculating that it would cost him $31 025 to buy an 85¢ hamburger daily for a full year (365 days). The question was: What would you say to him if he told you that? Nine of sixteen students agreed that $31 025 was correct after they had an opportunity to calculate. Of course they multiplied 365 X 85 = and saw the correct sequence of digits. The other students saw that something was wrong. Two students, F-111 and M-124, both high-achievers in mathematics (of five placed in this category), were the only ones who immediately saw that thirty-one thousand dollars was unreasonable for 365 days at less than one dollar per day. The other five who said this was wrong concluded this by doing the actual calculation and then noting that it was a decimal placement problem in stating the result as \$31 025 rather than \$310.25. Surprisingly, one high-achieving student, M-113 (second to M-115 in mathematical ability), did this calculation and agreed that \$31 025 was correct.

If estimation ability and judgment of reasonableness of answers are considered to be characteristics of good calculator users then this is so for only the high-ability students. Even then it is not always true that such students are alert to this problem. In spite of the large amount of practice exercises students did on these learnings, the skill did not transfer readily to other related situations.

Most problems in calculator application appeared to have a mathematical source -- conceptual, algorithmic, or hueristic. A common illustration of this is the fact that students were reluctant and unlikely to analyze problems before resorting to calculation. It was assumed that such analysis would be characteristic of efficient calculator users. In doing multi-step problems students would calculate at intermediate stages and use these figures in proceeding to the next stage.

A typical example of a problem where students were anxious to calculate intermediate steps is illustrated by the last question on the "Powers" page (Appendix C). The last part of the question required that a total of 1.8447×10^{19} kernels of wheat be evaluated at $90 per tonne (metric ton). The number of kernels had already been established from previous calculations. At both schools when it had been established that the number of kilograms was $\frac{1.8447 \times 10^{19}}{147\ 000}$ (since it was known that about 147 000 kernels make one kilogram), students were anxious to do this calculation before proceeding to the next step. The class and instructor then established that this quantity of kilograms would be divided by 1000 to convert to tonnes and finally the value would be found by multiplying by 90 so the final result was $\frac{1.8447 \times 10^{19}}{147\ 000} \div 1000 \times 90 =$. It was in such situations that students would calculate intermediate values rather than display on paper the complete sequence of calculations.

The above example also illustrates another characteristic of good calculator users. This is the habit of writing out a sequence of calculations before actually doing them. Students were reluctant to write any steps. The predominant pattern was to read the question and immediately calculate. The lack of a written record made checking more difficult. A written sequence made the analysis much easier to follow and generally led to fewer errors. The researcher always encouraged this and displayed the format when doing sample problems. However, this pattern did not become a regular habit of the students. Often the students were asked to supply the KSS first and then to do the calculations.

The writing of the KSS was another characteristic of good calculator users. However, students would not normally do this. The availability of a machine apparently led to a disregard of previously-taught mathematics procedures. Since the machine was available that was the thing to do. Students suddenly attempted to use the machine to the exclusion of any other methods which would support the machine. When students were asked to give the KSS (and this was usually done for complex problems), they tended to be weak in this ability. The high-ability students could usually do better at stating a correct KSS. Low-achieving students would confuse the KSS with a statement of steps used in solving a problem. Also they tended to indicate partial answers in the KSS. For example for

$\frac{1.8447 \times 10^{19}}{147\ 000}$ ÷ 1000 X 90 the KSS would probably be 1.8447 EE↓ 19 ÷ 147 000 ÷ 1000 X 90 = . Student F-116, who was not proficient in doing complex sequences, would likely show 1.8447 EE↓ 19 ÷ 147 000 = 1.2549 X 10^{14} ÷ 1000 X 90 = . This student would record the intermediate result, 1.2549 X 10^{14} .

From the data gathered here it is apparent that the following practices of good calculator usage are not developed in an incidental manner:

--the habit of constantly glancing at the display

--assessing reasonableness of answers

--estimating approximate results

--analyzing problems and displaying a full KSS prior to calculating

--distinguishing between a KSS and the mathematical steps as well as the intermediate results.

Q16--How well do calculator students use the calculator?

This question is intended to give a general overview of the ability of grade 8 students with calculators.

As a trivial point it is noted that all students learn very easily the general overall calculator operation. They easily learn the general pattern of key sequences for basic calculations. Questions such as 79 + 273, 897 - 648, $^{-}247 \div {}^{-}43$ are easily carried out by all students. When chain calculations occur which require a direct feed-in students are able to do these without difficulty. Such an example is $275 + {}^{-}34 + 63 \times {}^{-}376 =$. This is relatively easy for students because the calculator is programmed to compute with the correct order of operations. Such a computation as 835 X (89 - 64 + 38) would also present no difficulties because the calculator contains the parentheses function and students would feed these in. The calculator would then give the correct result. When computations involve complex fractions, for example $\frac{876}{38 + 74}$ or a more complex one such as $\frac{876 \times 16 + 77}{38 + 74 \times 9}$, only the high-ability students would likely get this right. The average and low-ability students would most likely make an error in dividing numerator by denominator and get the result for $\frac{876 \times 16 + 77}{38} + 74 \times 9$.

More details of how well students cope with various mathematical calculations are provided in previous questions (see Q4, Q5, Q6, Q7, Q10, Q13, and Q14) and in Chapter V (Case Studies).

All students developed efficient techniques in manipulation of the calculator. Most either operated it with the non-writing hand or alternated the writing hand from calculator to paper (for recording results). One student used the two-thumb technique or alternated this with the use of the writing hand, but she tended to make errors because of her carelessness.

Generally students did not watch the display sufficiently. This, coupled with their lack of use of estimating skill and lack of skill in assessment of reasonableness of answers, contributed to further errors. The number of errors was usually dependent upon academic ability, the high-ability students making fewer errors of this nature than the students of lower ability.

Students do not analyze problems. As a result they have a difficult time with problems needing several steps or with computations requiring several stages. Students of high ability have better success than students with low ability, probably because they can decide on the steps required and can retain these mentally. Students of low ability do not have this capability and so are not able to solve these problems. It appears that the thinking of students follows a pattern, for example, "I'll add these numbers and divide the result by this number [and they do this on the calculator]; now I must multiply this by the next number [entering this next step into the calculator] and that's my answer." If this analysis could be accomplished in written fashion by low-ability students, it is possible that their success rate would be greatly improved.

Finally, students are reluctant to write a KSS for computations. This is closely related to analyzing a problem, for if the KSS could be stated correctly, it would mean that the problem had been properly analyzed and that the answer would be correct. The writing of the KSS would need to become a learned skill as part of the development in calculator usage.

Research Questions Related to Student Attitude and Interest

Q17--How does the attitude of the calculator students change over the period of the project?

The attitude of students was checked by a questionnaire (Opinion Check) given to students early in the project and then given at the end. The mean of the first check was 35 and that of the second was 34.0625. The standard deviation had increased slightly from 7.31 to 8.00. For the non-calculator users in the two classrooms (N = 34) the mean of the Opinion Check went from 30.62 to 27.85, a decrease of 2.77. The standard deviation changed from 9.57 to 9.83. Therefore the change was in the same direction for both groups although the mean change was a larger decrease for the non-calculator users. From this evidence it is clear that the use of the calculator did not significantly alter student attitudes to mathematics. An inspection of the actual scores for the calculator students, Table II, indicates how slight the changes were for individual students. The most dramatic changes were scored by F-114, M-122, M-123, and M-125.

One major increase was for M-122. He seemed to increase in motivation throughout the project and this may account for the positive change. Another was for M-125, an average mathematical achiever, who responded positively throughout the project and seemed to get a great deal of satisfaction from his ability to answer mathematical questions using the calculator.

The most spectacular change was a negative one of 18 points for M-123. This student had better than average ability. He was generally cooperative but not one who liked to work diligently at calculator operations. This student usually sat back and played aimlessly (with a pencil, a pen, the ruler, the calculator, or whatever was handy) and waited for others to do the problems. When the researcher made it obvious that he was waiting for the student to do a calculation he would do it. If a particularly difficult problem was posed this student's approach was spontaneous. That is, he took no time to consider the problem or to analyze for a possible approach. He simply blurted out one answer after another seemingly to attempt to hit the correct one in that manner.

TABLE II

Pretest and Posttest Scores on Student Opinion Check

Student	Pretest Scores	Posttest Scores
F-111	26	24
F-112	27	37
M-113	38	33
F-114	25	17
M-115	37	41
F-116	38	35
M-117	34	35
M-118	33	32
F-119	47	45
F-120	37	35
M-121	27	26
M-122	32	42
M-123	47	29
M-124	45	43
M-125	28	37
F-126	39	44

There is no explanation for this student's dramatic negative change in attitude. Throughout the project there was no indication that he might be changing in his feelings about mathematics. His teacher reported no changes. The researcher noted that he seemed to respond the same way throughout the project.

In summary, students' attitudes towards mathematics did not change throughout the calculator project. Only a few individual changes could be considered significant. It is not possible to determine the reasons for such changes.

Q18--Will student interest in the use of the calculator be high? Will interest continue without further stimulation by the researcher at the conclusion of the project?

The calculator is a most highly motivating device. Student interest was high at the beginning of the project. At both schools the researcher had a number of requests from other students asking whether they could get into the project; this was after the project had been in progress for about two weeks. Students do not view the calculator as simply a machine helping them because it removes the drudgery of calculation and thereby helps them tackle more difficult and more interesting problems. Rather they view it as an end in itself, a device that does the work for them. Therefore interest is high in using such an aid.

A student survey of the Calculator Students was conducted at the completion of six weeks of the twelve of the project. At this point seven of sixteen students stated that they regularly used the calculator for other subjects. The subjects were mainly Science but Social Studies was also mentioned. Nine students, including the five who said they used the calculator for other subjects, stated that they regularly took the calculator home. Three of the nine said they did this so that the calculator would not be stolen. Six said they used it for mathematics or other homework and two of the six said they used it for other things at home (games, school science project, other calculations--unspecified).

Students did not typically carry their calculators daily with their books as they went from class to class. Three said they did while thirteen said they did not. However, normally the students would pick up their calculators from their lockers just prior to mathematics lessons. It is interesting to note that the best calculator-user (and the best mathematics student), M-115 of School A, replied that he often used the calculator in other subjects, normally took it home and used it in a variety of situations (projects), and generally carried the calculator with him to other classes. The best calculator user for School B, M-124, had the same responses to the questions; that is, he used the calculator in other subjects, normally took it home, and generally carried it with him to other classes throughout the school day.

In the six weeks follow-up students were asked if they had used the calculator since the project had concluded. Fifteen replied that they had while one said he had not. The extent of use was not specified in the question but in verbal requests for clarification the students were told that if it was generally their practice to use it then they were to respond positively. When asked if they took the calculator to mathematics classes every day in the interim, seven responded positively. However, the qualifier "every day" led some to say "no" to this question but then add that on occasion they did take them. All sixteen responded that from time to time they had used the calculator. Five said they used it in science but all mentioned mathematics in such topics as Rate, Ratio, Percent; exponents; general calculations; decimals; equations; and fractions. One student said she used it in a Business Education option for calculating percentages and in completing bank forms. Twelve of the sixteen students said they had used the calculator at home over the six weeks. The most common use was for doing homework (mathematics most often but also science and business mathematics). One student, the one of highest ability, stated that he used it to calculate grocery lists and for calculations in an electronics project.

The use in the interim may be thought of as low. However, if it is recalled that the teachers did not make a point of adjusting the curriculum nor of making any special efforts to acknowledge the presence of calculators nor to accommodate them in a special way, then it means that the interest of the students was well-maintained. Another good indication of the interest is the fact that twelve of the students purchased their calculators after the follow-up session, when they were given this opportunity at 50% of the cost. One calculator was lost (the student had intended to purchase it) and three others were returned to the researcher. One student purchased two calculators. Of the three returned, two said they already had calculators at home while the third said he would like to purchase it but was saving his money for the coming holidays.

Research Question Related to the Curriculum

Q19--Which topics of mathematics lend themselves to calculator application and which do not?

During the project students were involved in two units, one on Rational Numbers and the other on Rate, Ratio, and Percent. In the unit on Rational Numbers students also worked with integers. A unit test (see Grade 8 Mathematics, Appendix E) was given as a regular test in School B and students were not to use calculators. The researcher then gave the test at a later date to the Calculator Students at both schools. Both units involved a number of sections dealing with solving of equations such as $2.7x + 38 = 49.61$. A consideration of the results on the Grade 8 Mathematics test, which was on integers, shows that such work is very easy on the calculator. The results are shown in Table III. This table gives the number of students making a given number of errors. The mean number of errors was 2.33. The table gives the results on only the first thirty questions, which are direct calculation questions. Questions 31 to 40 deal with solving of equations and with evaluation of expressions which require some additional skills or knowledge. Of the total of 31 errors made on 450 questions (30 items for each of 15 students), eight of these errors were errors of sign. That is, the answer was numerically correct but had the incorrect sign. Student M-125, an average achiever, made eight errors but four sign errors. It is surprising that the lowest achiever, F-120, had no errors while another student, F-114, an average achiever, had one error on the whole page (all 40 questions) and this was

TABLE III

Results on the Grade 8 Mathematics Test

Number of Errors	Number of Students
0	4
1	4
2	1
3	4
5	1
8	1

a sign error. It is reasonable to conclude that direct questions of calculations involving integers are particularly applicable to calculator computation.

In considering questions 31 to 40 of the test (solving of equations and evaluating expressions) the results show that these are not so easily done. Of the 150 questions, 31 were done incorrectly while 14 were omitted for a total of 45 or 30% error. However, 33 of the 45 errors (or about 75%) for these questions, were made by six of the fifteen students. The four highest-achieving students of the fifteen (the missing student was one of the top five) made only five of the 45 errors.

The most dramatic effect of using the calculator for this test was in the time used. The teacher allowed a forty-minute class period for the test. Most students used the full time. With the calculators the longest time used was 22 minutes with most time spent on the last ten questions. For the basic calculations most students would have been finished in about twelve minutes. In fact the fastest time for the 40-item full test was twelve minutes and several were finished in fifteen minutes and some in eighteen minutes.

There is no doubt that for calculations involving integers the calculator makes a dramatic difference not only in the time it takes to do such calculations but also in the degree of accuracy. If students were taught algorithms which would be of application to calculators for solving questions then the effects on such a test would be still greater.

When students were confronted with calculations for regular fraction questions they had reasonably good success using the calculator to assist them with the steps required to obtain an answer in fraction form. Of course the calculator is much more useful when a student, faced with something like $19\frac{7}{12} + 13\frac{14}{19}$, is permitted to obtain the answer in decimal form. As indicated previously, Q8, students would prefer working in decimal notation rather than in fraction form when using the calculator for fractional computations.

Students generally found the questions calling for calculations with powers extremely easy using the calculator. The calculator used for the project, the TI-30, has a y^x key which raises a number, y, to the power, x. For instance $\underline{2\ y^x\ 12\ =}$ is the KSS which will raise two to the twelfth power. However, it will not raise a negative number to any power; attempting such a calculation results in the display of 'Error'. Whenever the topic of power calculations was raised no difficulties were experienced in calculations. The troubles students encountered stemmed from their lack of knowledge concerning powers. Questions such as 3^{2^5} presented some difficulty in interpretations. Series such as $1 + 2 + 2^2 + 2^3 + \ldots + 2^n$ also led to more errors, not only because of the level of difficulty, but also because of the number of separate steps required. In one problem students were required to calculate this series to $n = 63$. Only a few could successfully complete it simply because the 252 moves needed left too much opportunity for errors to be made.

Generally students did not seem to find the calculator of great use in solving equations. One reason may be that a specific format was required and this coupled with the simplicity of the numbers involved did not present opportunities for using the calculator. Only the high-achieving students (the top five) who would take intermediate steps more easily could do equations quite readily. Even such simple questions as $3n + {}^{-}2 = {}^{-}17$ led to some difficulty for many students but the good students probably did these by recognizing that from -17 it is necessary to subtract -2 and then divide by 3. On the other hand if a student is required to go through a sequence as illustrated following, there is no point in using a calculator for such simple problems:

$$3n + {}^{-}2 = -17$$

$$3n + {}^{-}2 - {}^{-}2 = -17 - {}^{-}2$$

$$3n + {}^{-}2 + 2 = -17 + 2$$

$$3n = -15$$

$$(\frac{1}{3} \times 3)\ n = -15 \times \frac{1}{3}$$

$$n = -5$$

If students could be taught appropriate algorithms, solving of equations at this level would be quite a simple matter.

Research Question Related to Parent Attitudes

Q20--What are the attitudes of the parents of calculator users towards the use of calculators?

At the end of the project the fifteen parents (two students in the project were from the same family) completed the Opinion Check for parents (see Appendix E). This was an attempt to get the opinion of parents about the use of calculators in grade 8 mathematics. No indication was given which parent completed the check or whether it was a joint effort. Table IV summarizes the results of this check.

Twelve of the fifteen homes have calculators and in all twelve, students are allowed to use them. Five parents made comments about restrictions. One said, "To be used for checking mathematics calculations and not doing actual questions." Another said, "I

TABLE IV

Results of Parental Opinion Check

Question Number	Number of Responses in Categories Shown				
	Yes	No			Omitted
1.	12	3			
2.	12	1			2
3.	13	2			
	Hinder	Improve	Little or No Effect	No Opinion	
4.	5	9	1		
5.	3	6	3	1	2
6.	3	8	4		
	Highly Dependent	Dep. to Slight Degree	Not Dependent		
7.	2	9	4		
	Yes	No			
8.	10	3		2	
9.	4	8			3
10.	7	5			3
11.	15				
	Grade 10	Grade 8	Grade 7		
12.	2	3	10		

believe they should not be used when it is for work they should figure in their head." A third said, "They are not to be used for math homework," while a fourth stated, "Not for doing homework where we have felt she should use her head." One parent simply replied, "None."

Only two parents did not favor the use of calculators "To some extent in Junior High School." Although parents have calculators at home and students are allowed to use them, the parents are not so convinced that the use of calculators will be so helpful. Although nine parents said the use of calculators would improve the children's performance in basic computational skills, five felt their use would hinder this development and one felt it would have little or no effect. The division for number 5 is greater since six parents felt the overall performance in mathematics would improve, three said it would be hindered and three said it would have no effect; one had no opinion and two omitted this question. Eight parents felt the attitude of students would improve. Nine parents felt that students would be "dependent to a slight degree" on the calculator in performance of basic skills; two felt the dependence would be high and four felt they would not be dependent.

In spite of a somewhat divided opinion on the effects of calculators on students' basic skills, general mathematics or attitudes, parents were more certain that calculators should be permitted in the grade 8 classroom. Ten felt they should be, three said they should not and two had no opinion. However, parents are stronger in the opinion that they should not be used in all subjects; eight felt that way while four supported use in all subjects and three omitted this question. In the matter of using the calculator for tests, parents are not so clear. Nevertheless, it is perhaps surprising that seven favored their use, five said, "No" and three omitted this question. All fifteen parents agreed that the School Board should provide instruction to the students in the use of calculators and this should happen in grade 7 according to ten parents while three favored grade 8 and two suggested starting in grade 10 (Senior High School).

In summary it appears that parents are not definite in their opinions about the use of calculators. They have calculators at home and allow students to use them and favor their use to some extent in Junior High School. On the other hand there is some suggestion that the use of calculators will hinder performance in basic computational skills or in mathematics. Certainly parents are of the opinion that the calculator will make students dependent at least to a slight degree on calculators. On the other hand parents favor the use of calculators in grade 8 but not for all subjects. They also favor the use of calculators on tests. They are unanimous in the opinion that instruction should be given in the use of the calculator.

The Opinion Check invited parents to make any comments they wished about the use of calculators; six did so. The following are five of the comments:

> If a minicalculator is going to be used in a school I think it should be limited to an extent. A student should make use of his own power to think and solve problems. I do believe in the use of them, however. I also believe a certain amount of instruction should be given where they haven't used a calculator before.
>
> As long as they have knowledge as to how mathematics works there should be no drawback in the use of the calculators, but it should not be used as a substitute.
>
> I'm just concerned that the calculator will not force the student to use the times tables and thus they will forget in time.
>
> I do not believe calculators are beneficial or necessary in grade eight, especially in math classes.
>
> I feel that in the elementary area of education, not enough emphasis is being placed on basic computational skills up to the junior high level. If this emphasis is not placed early in a child's education, I feel the child's brain will become lax if allowed to depend on agencies such as calculators.

The sixth parental comment is lengthy. The parent stresses two points: (1) Arithmetic skills are necessary but this parent feels students are not yet well enough prepared in these before entering junior high school. One skill mentioned is the ability to estimate, in which students are poorly prepared. (2) Mental arithmetic is stressed but this parent feels not enough practice will be experienced when calculators are used. The variation in calculators also makes this parent wonder about how students can adjust. Finally this parent says:

> However our children are products of the computer age and should have an understanding of these tools. I tell our kids, however, that they have a splendid calculator always with them. It's called a brain.

Research Questions Related to Further Changes in Students

Q21--What changes occur in the calculator students' knowledge of mathematics concepts and in problem solving ability as measured by the Canadian Test of Basic Skills?

The Canadian Test of Basic Skills (CTBS) contains two mathematical subtests, one which tests basic mathematical concepts and the other mathematical problem solving. A sample question from the former is

92. What numeral should replace n in the number sentence $\frac{3}{4} \times n = \frac{2}{3}$?

1. $\frac{1}{2}$ 2. $\frac{5}{7}$ 3. $\frac{8}{9}$ 4. $1\frac{5}{12}$

and a sample of the latter is

86. Connie bought 2 record albums at $3.95 each and 6 individual records at 89$ each. How much in all did Connie pay for records?

1. $5.34	3. $13.24
2. $8.79	4. (Not given).

The CTBS was given both as a pretest at the end of January and again as a posttest at the end of April. The results are shown in Table V. The data provided for these tests give conversion of the raw scores to Grade Equivalents and if desired these can be converted to Percentile Ranks. The average GE (Grade Equivalent) for the concepts tests went from 93.5 to 98.19, an increase of 4.69 points. This increase is significant at the 5% level using a one-tailed t-test. You would expect that each student would add three points to his GE and for the concepts test an inspection of the scores shows the increases (and decreases) and where these occurred. The mean on the problem solving test went from 88.56 to 92.94, an increase of 4.38 points. This increase, however, was not significant at the 5% level. An inspection of the pre- and posttest scores shows a wider fluctuation. Although there are many spectacular gains (13, 14, 15, 18, and 27 points), there are several decreases (two of 9 points and three of 4 points).

The posttest was done without the use of the calculator. The average GE score of the Calculator Students rose more than the expected three points.

TABLE V

Grade Equivalents for Calculator Students on Canadian Test of Basic Skills

Students by Code	Mathematics Concepts Pre-test	Mathematics Concepts Post-test	Mathematics Concepts Change	Mathematics Problem Solving Pre-test	Mathematics Problem Solving Post-test	Mathematics Problem Solving Change
F-111	113[1]	106	-7	97	101	+4
F-112	77	79	+2	80	95	+15
M-113	110	108	-2	97	111	+14
F-114	94	103	+9	99	95	-4
M-115	110	119	+9	88	95	+7
F-116	85	94	+9	68	86	+18
M-117	98	105	+7	77	90	+13
M-118	91	103	+12	95	86	-9
F-119	92	96	+4	90	88	-2
F-120	70	70	0	46	73	+27
M-121	74	91	+17	103	99	-4
M-122	96	98	+2	90	92	+2
M-123	92	92	0	103	99	-4
M-124	96	101	+5	97	99	+2
M-125	100	100	0	90	90	0
F-126	98	106	+8	97	88	-9

[1] A Grade Equivalent of 113 means that this student's score is the same as the average of students in the third month of the eleventh grade of the norming population.

However, there was no consistency when individual scores are considered.

Q22--How did calculator students change in basic knowledge of computational facts (addition, subtraction, multiplication, division)?

The students were given one-minute tests on the basic facts of addition, subtraction, multiplication, and division (see Appendix E). These tests contained a large number of basic facts and the students were instructed to complete as many as they could in one minute. The number of items correct constituted the score. The same tests were used at the beginning of the project and again at the end. Table VI gives the means both before and at the end of the project for the Calculator Students and for their classmates, the Non-calculator Students.

TABLE VI

Means on the Basic Facts Checks

	Calculator Students		Non-calculator Students	
	Pre	Post	Pre	Post
Addition	42.1875	47.1875	40.1471	44.4706
Subtraction	30.9375	35.375	27.4706	31.5882
Multiplication	36.75	40.3125	35.7059	36.3824
Division	37.25	42.5625	31.5588	36.6765

None of the differences between means is significant at the 5% level between the Calculator and Non-calculator students (using a two-tailed t-test). An inspection of the means for the Calculator Students shows that for all four tests there is an increase from the pre- to the post-periods. This is also true for the Non-calculator Students. All except one of these differences, the multiplication check for non-calculator students, are significant at the 5% level using a one-tailed t-test.

The above indicates that at least for the three months duration of this project the calculator students progressed as well, and even slightly better, than the non-calculator students. Certainly they did not decrease in their ability in the recall of the basic facts of arithmetic.

Q23--How did the calculator students change in knowledge of Rational Numbers?

Both schools in this project used the consortium materials prepared by provincial teachers and used in many schools in the province of Alberta. Rational Number test items based directly on this curriculum were prepared (see Appendix E). Two very similar tests were developed to be used as pre- and posttests. Students were given one class period, forty minutes, in which to write the tests. For the pretest no calculators were used but in the posttest the calculator students were allowed to use calculators. Table VII gives the means for both groups of students for the Rational Numbers tests and for the Rate, Ratio, and Percent tests.

TABLE VII

Means on Rational Numbers Tests and on Rate, Ratio, and Percent Tests

	Calculator Students		Non-calculator Students	
	Pre	Post	Pre	Post
Rational Numbers	17.75*	27.0625	15.2941	17.8529
Rate, Ratio Percent	10**	29.125	9.2059	23.4118

*Maximum for pre- and posttest was 54

**Maximum for pre- and posttest was 53

In the Rational Numbers Pretest the difference between means of 17.75 (Calculator Students) and 15.2941 (Non-calculator Students) was not significant at the 5% level. However, the corresponding posttest means were 27.0625 and 17.8529 and the difference was significant at the 5% level.

The conclusion is that the Calculator Students did significantly better on the Rational Numbers Posttest than did the Non-calculator Students.

Q24--How did the calculator students change in their knowledge of Rate, Ratio, and Percent?

Since it was anticipated that students would be working on the unit on Rate, Ratio, and Percent, both classes were given a pretest on this unit. This test was based on the consortium materials just as was the Rational Numbers test. A similar test was constructed to be used as posttest. The means for this test are included in Table VII together with the information for the Rational Numbers Tests. The difference between the pretest means was not significant at the 5% level nor was the difference between the posttest means. Students were allowed to use the calculators on the posttest. The posttest value of the mean for the Calculator Students is larger by 5.7 points but this difference is not enough to be statistically significant. School A had completed the unit on Rate, Ratio, and Percent at the time of the testing, but School B had not. Table VIII shows the means of calculator and non-calculator groups at each school. The difference at School B is more pronounced and this contributed more to the combined means than did the smaller difference at School A. It appears that the calculator students had a decided advantage at School B where the unit had not been completed.

TABLE VIII

Posttest Means on Rate, Ratio, Percent Test

	Calculator Students	Non-calculator Students
School A	30.75	27.625
School B	27.5	19.6667
Schools Combined	29.125	23.4118

Research Questions Added During the Project

Q25--What general mathematical knowledge or skills appear important in calculator applications?

Based on experience with both calculator groups for a total of slightly less than thirty-six class periods for each class, a number of observations were made which deal with the mathematical behavior of the students but which are not necessarily calculator-oriented behaviors. Several of these observations are given in order that they may assist others in pinpointing trouble spots in grade 8 mathematics.

One general area of concern relates to division. Many students (and not always the low-achievers) experienced much confusion in handling the division sign (÷) as it relates to a fraction, $\frac{A}{B}$, and in verbalizing as to whether one says "A divided by B", "B divided by A", "B divides into A", "B goes into A", and other possible interpretations. Although this occurred many times in class, it was also observed in personal interviews. For example when faced with the problem of converting $\frac{7}{8}$ to a decimal a student wrote 8 ÷ 7 with the explanation that, "I'm dividing 8 into 7." Perhaps a standard format for A ÷ B should be adopted such as "A divided by B" and this same form should be applied to $B\overline{)A}$. The following excerpts will illustrate the difficulties encountered:

Excerpt 6 (M-117):

[In this question the student is required to change $\frac{430}{23} = \frac{}{322}$; that is, $\frac{430}{23} = \frac{}{322}$?]

S: You divide 23 and 322 [Entered 23 ÷ 322.]

R: Push the = key and look at the answer.

[Student did so.]

What did you divide?

S: 23 divided 322

R: Divided what--into or by?

S: By.

R: Yes. So, do you want to divide 23 by 322?

S: No.

Note the uncertainty as to the order (student first entered 23 ÷ 322) and then the lack of preciseness in the language to be used in stating the division. The following excerpt further illustrates the lack of clarity in the words used:

Excerpt 7 (M-118):

[The student was considering the possibility of reducing $55\frac{22}{133}$.]

R: Are you finished?

S: Do you want it in lowest terms?

R: Yes.

S: That [pointing to 22] goes in twice by 11.

In connection with the same situation, $55\frac{22}{133}$, the following occurred with another student:

Excerpt 8 (F-119)

R: Now is that in its simplest form?

S: Well, it might go into 11 [pointing to 133], I'm not sure.

The word "divides" is not used in its proper meaning where "A divides B" means that B = AX + 0 and X is an integer. Students use "go" for this meaning. The next excerpt for the same student points this out:

Excerpt 9 (F-119):

R: You said divide 133 into 11 [but 133 ÷ 11 was required].

S: Oh! [Did 133 ÷ 11 = .] No, it doesn't go [meaning that 11 does not divide 133].

R: So?

S: So that's the answer [meaning $\frac{22}{133}$ is in lowest terms].

R: Why?

S: Well, if 2 doesn't go into anything ..... well .... 2 doesn't go into anything that has a 3 at the end.

Another somewhat related area since it refers to language is in verbal statements about multiplication. Students often said they were "timesing" for multiplicative situations. In both schools this language was used. One teacher used that term often in talking about multiplication. The term would also appear in such instances as, "you times by 12", when the meaning is that the number is to be multiplied by 12.

Excerpt 10 (M-118):

R: It says 22 X 3 + 4.

S: Well, you times it first and then add.

R: When you feed 4 + 6 X 3 = into the calculator, what does the calculator do first?

S: Timeses it.

Students should be given better guidance in the proper language of division, multiplication, subtraction and perhaps even addition. Should we say "18 minus 6" for 18 - 6 or "18 subtract 6" or "6 from 18"? The language is not the prime target but rather the uncertainty of the meaning. Perhaps a more standardized form of language would help in establishing the meaning.

In another category it was noted that students have poor understanding of the equality or inequality of certain quantities. For instance they are not certain whether $A \div \frac{B}{C}$ is the same as $A \div B \div C$, $\frac{A}{B \times C}$ is the same as $A \div B \div C$ or as $A \div (B \times C)$, whether $A - (B + C)$ is the same as $(B + C) - A$ or as $A - B - C$. In fact the equality of $\frac{A}{B \times C}$ and $A \div B \div C$ really surprised students; it seems that they could not grasp the sudden disappearance of a multiplicative situation to be replaced by a division process.

Students generally had a reluctance to carefully analyze problems. It seemed more typical of the students to rush in and begin some kind of calculations. One question posed to the students by the researcher asked them to give the number which would represent the side of a square whose area was 1000 km^2. Even though students were cautioned that this is not as simple as at first it appears (the usual first response was 100) and even though the researcher told them that it is not a whole number, students had a hard time grasping the square root application. They had demonstrated earlier that they knew the basic meaning of square root but they could not analyze this easily. One student, M-123, who had the potential of being good with the calculator, never spent any time analyzing and often shouted out one answer after another or one suggestion after another as to what should be done to arrive at a solution.

Excerpt 11:

R: If I have a patch of land that is square and contains 1000 km^2, can you tell me how long and how wide that patch must be?

M-123: 1 million, 1000 by 1000

R: I want to know how long and how wide it must be so that the area is 1000.

M-125: 25

[Students were asked to do this on the calculator and write the answer. As the researcher went around he observed answers of 25, 250, 0.25, 1 000 000, 500, and 4000.]

[Many guesses were made and students seemed disappointed that they were wrong, yet noone thought of checking and deciding in that way if they were right or not.]

M-123: 500?

M-122: 100?

R: No.

[M-123 kept insisting on 100 because he said 100 X 100 = 1000.]

M-122: Is it in the 100 category?

R: You should know it when you get it because you can always check. It's not a whole number; that's a hint.

M-123: 2.5?

R: No, 2.5 is not involved in it at all.

M-124: 33.33 333. [He had divided 1000 by 3.]

[Finally M-124 got the answer and when the researcher asked him how many keys he had to punch after entering 1000 and his answer was one, a number of students then seemed to hit on the correct answer.] [The answer was then provided for all those who were not able to get it, including M-123.]

M-123: Oh, I had that before but I erased [cleared] it.

Although M-123 was the prime example of a student who rushed into questions with little or no analyses, many other students behaved similarly. There was a definite reluctance to establish a step-by-step pattern. See the next excerpt for a further illustration of attempts at analysis as carried out in the classroom.

Excerpt 12:

[Students were presented with a problem similar to the Tower of Hanoi problem but having six discs.]

R: If I had one of these [towers] with 6 discs on it and then had two other empty pegs [towers], how many moves would it take to move all 6 from one peg to some other one? Remember, the rule is that a larger disc can not go on top of a smaller one.

M-123: 6

[As a class the problem was done moving three of six discs and it was observed by all that seven moves were needed.]

M-123: So it'll take 14 to move the rest.

R: But you're making a rather hasty decision.

M-121: You have to move all of them round again and it won't work.

M-123: That's true.

R: How could you attack this problem and develop some kind of system? [No reply.] What's a good suggestion, something you could do whenever you are faced with a tough problem?

M-123: Get an easier problem.

R: Right. What would be an easier problem?

M-123: Just do three of them?

R: What would be easier yet?

M-124: One ring.

R: Good. [From here a chart for 1, 2, 3, ..., discs was developed to obtain a pattern.]

Related to the behavior of poor analysis was another characteristic which inhibited good mathematical behavior. This was the reluctance to write any intermediate steps, partial answers, a KSS, or perhaps a summary of the steps needed to solve a problem. The researcher attempted to set examples by blackboard work and by discussion but it is a habit of long standing and does not lend itself to such a quick change.

Related to the reluctance for analysis was a reluctance to state the computational steps for a whole problem before actually doing the computation. This was described under Q15 as an aspect of good calculator users.

Not only were students hesitant (or unable) to produce written steps needed in solving problems but their ability to verbalize a solution is very limited. The high-achieving students could explain quite well what was needed to solve a problem but in general this was a very limiting lack of ability. Most students replied in one or two words or in simple phrases or sentences. The researcher would need to question further and lead the group to the next stage. See Appendix J, Excerpt 6, for an illustration of a class discussion that shows the simple answers and the step-by-step discussion.

Students typically display a lack of checking work. Many of the errors made would be eliminated through checking. In the Grade 8 Mathematics Test on integers, for instance, students would do $-6 + n - -3$ and get $n = 9$. A quick check would eliminate such an error. Even though this has been a universal complaint of teachers, the presence of calculators should provide teachers a good motivational device which would stimulate checking. Although the example illustrated would not require a calculator for solving the equation nor for checking, for more complex equations it would be extremely useful.

At times students appeared to lack necessary mathematical knowledge for efficient calculator use. For example, in checking for factors of 257 students would try 2, 3, 4, 5, 6, 7, 8, ..., not realizing that composites of previously tried numbers need not be tried. Similarly, they could not accept $\sqrt{257}$ as the largest necessary test number. Perhaps some of these concepts are too sophisticated for grade 8 students, but they are in the curriculum and are taught at this level (as well as in grade 7). Wide discussion with student verbalization should be employed to strengthen their understanding of such mathematics.

Q26--What might a teacher do to incorporate calculators into regular classroom mathematical lessons?

From observations made it is obvious that some activities are taking place which bring calculators into the classroom. School A has a classroom set of simple four-function calculators and these have been used by some teachers. The use is in separate calculator lessons where the activities are designed to use the calculator in order to develop certain skills in junior high school students. These activities are typical of many suggestions for calculator use found extensively in the literature (see Appendix F for sample questions posed). These lessons were usually separate ones at the end of the formal curriculum for that grade. However, from this project it is possible to make a number of suggestions which teachers should find helpful.

Teachers should become familiar with calculators so that they could appreciate their potential as well as understand their limitations. Teachers should encourage students to purchase calculators and should give them guidance in their selection. This is important since the market is being flooded with many kinds of calculators, many of them poor purchases in terms of cost for the functions available. The light-emitting diode (as opposed to the liquid type) with a rechargeable battery pack is still probably the most economical and the most versatile. For under $30 a student can obtain a good, scientific calculator that would serve a student throughout post-secondary education.

Students should be given problems with the calculator in mind. For instance, problems that deal with room dimensions could be based on actual measurements of rooms taken by the students. "Easy" numbers are not necessary as the calculator work is not much more difficult when numbers are more complex. Calculators should be incorporated into the regular mathematics lessons. Many problems and assignments would need to be adjusted so that they become suitable for calculator application. Instruction in calculators would need to be given and special calculator lessons would need to be planned.

Students should be allowed to do more common fraction computation in decimal form. However, teachers will probably insist that students know the pencil and paper algorithms so questions with "nice" fractions, such as $8\frac{1}{2} - 2\frac{2}{3}$ will form part of the core. However, calculator students should be instructed that the algorithm could be $\underline{8 + 1 \div 2 = -(2 + 2 \div 3 =}$ (at least on a TI-30 calculator). They should be required to round off answers to some reasonable point such as to the nearest hundredth. Perhaps this would mean that the algebraic study of rational numbers would be postponed until students do a more formal study of algebra in later grades. This seems reasonable since they do not do this under present circumstances in grade 8.

Teachers must recognize (and accept) that calculators make trivial such computations as 0.001 27 ÷ 0.012. The objectives of work in this area could be altered. For example, in questions like this students might be required to estimate the result or to state where the answer will fall by giving limits such as $0.1 < X < 1.0$ or is it $1 < X < 10$.

More time must be spent on analysis of problems but fewer problems must be posed and assigned. A complete analysis should precede any calculations.

Examples: (1) Calculate $\frac{75 + 17 \times 35}{16 \div 64 + 32}$.

The discussion might center around the meaning of the exercise, the order in which one might compute, how this would be keyed into the calculator (with a written KSS), and how the calculator will execute the steps to arrive at the solution. Perhaps most important, the students should suggest estimations before they calculate.

(2) An automobile trip consists of sections of 850 km, 638 km, 747 km, and 878 km. Find the total cost of gasoline at 83.9¢ per gallon for this trip if the car averages 33 km/gal.

Students should be encouraged to set up the solution as $\frac{850 + 638 + 747 + 878}{33} \times 83.9 =$ and then

execute this with the following KSS:
850 + 638 + 747 + 878 = ÷ 33 X 83.9 = . The result would then be recorded as $79.15. If the separate total number of kilometres is required this could be noted and recorded when the first "=" sign is pressed. Similarly if the number of gallons is called for, this could be noted when the "X" sign is pressed.

The discussion for this problem could centre around the difference between using 83.9 and 0.839 in the calculation and perhaps why $79.15 is reported in view of the actual result obtained (7914.5667).

CHAPTER V

THREE CASE STUDIES

In the previous chapter some of the answers to the research questions provide information on the thought processes occurring as the students attempt solutions to problems. However, in order to illustrate more clearly and in greater detail the ability of the students in the use of calculators and to provide further insight into thought processes, the performance of three students will be described. The first is a student high in mathematics performance and in calculator usage, the second is average in these two characteristics, while the third is low. Each case study concludes with a summary of the strengths and weaknesses of that student in his ability with the calculator.

Case Study Number One: M-115

This student is a male whose age was 13-3 at the start of the project. His IQ is recorded as 135 Verbal and 123 Non-verbal on the Canadian Lorge Thorndike Test. His year-end grade 7 mathematics grade reported in the permanent file is 70 percent. This student is a pleasant, friendly person, able to articulate his position clearly.

He was obviously keen in his work with the calculator. From comments he made, he was interested in many different projects including a science one on which he was currently working. He disclosed that they had several calculators at home and that he had used one before; he did purchase the calculator used in the project when the opportunity was presented at the conclusion of the project.

Although he was interested in all types of unusual questions presented in class, he did not do much homework in order to complete worksheets. However, except for the last worksheets dealing with the calculations of π, he was able to complete them in class with some homework.

In using the calculator, this student's normal mode of operation was to use his left hand for operating the calculator keeping the right hand free for recording results. However, for some questions (like 1. b, of Checkpoint Number 6) he would switch to operating the calculator using both index fingers.

In working mathematics questions, this student applied his own strategies. This was obvious when his group was working on integers. When doing $42 + {}^{-}36 + {}^{-}8$ using a calculator, he did $\underline{36 + 8 - 42 =}$ and stated that he did it like that knowing that the answer would be negative and he would then insert the appropriate sign.

Excerpt 13 (M-115):

S: I did 36 + 8 = and then minus 42.

R: How do you know that's right?

S: Because 36 and 8 are both negative so you add them together and that's a negative number and then it's bigger than 42 so you subtract and call it a negative number.

R: In number 11 [${}^{-}316 - {}^{-}119 =$] you did 316 - 119.

S: So it's a negative.

R: What if it had been $+ {}^{-}119$, what would you have done?

S: I would subtract the two numbers.

R: You would subtract?

S: Well, if it was ${}^{-}316 + {}^{-}119$ then I'd add them and just take the negative sign.

This student was not using the +/- key (for changing the sign of the display) in the way that would be obvious. For ${}^{-}316 - {}^{-}119$ the KSS would normally be 316 +/- - 119 +/- = but this student employed strategies which he had previously learned, and applied these to his calculator strategies.

This student analyzed problems well. In the fifth week of the project his group was given the following to calculate: $\frac{75 \times 47 - 63 \times 12}{16 \times 14 - 12 \times 8} =$. Several efficient key sequences could be employed to solve this. Student M-115 did as follows:

75 X 47 - 63 X 12 = STO 16 X 14 - 12 X 8 = EXC ÷ EXC = .

For this point in the project it was a very good sequence. It is not the most efficient in terms of key strokes since the use of parentheses would be shorter, but nevertheless it was a good sequence.

This student was aware of certain requirements necessary in order to use the TI-30 calculator correctly. For instance pressing the "=" sign key completes all calculations to that point. This is a very important point and must be recognized in complex calculations. In finding the cost of fencing a city lot 33.5 m by 18.6 m at $13.95 per metre this student used the KSS 18.6 X 2 + 33.5 X 2 = X 13.95 = . He knew that no parentheses were necessary and that the first equals sign would complete the calculations and give him the perimeter.

In simple calculations this student did questions mentally. Questions such as 9 X 8, 18 + 7, 247 + 100, 8746 ÷ 100, and 2 476 345 X 1000 were done without using the calculator. In questions such as 768 + 768 + 768 + 768 he was quick to spot that one would do it as 768 X 4 = .

He understood the function of parentheses. One question he was asked to do was (86 X 86 + 47) X 25 + 17 = which he did as 86 x^2 + 47 = X 25 + 17 = .

Excerpt 14 (M-115):

R: Good. Now, if you forget the brackets and just did the question as shown (but left out brackets) would you get the same answer?

S: No, because it would go 47 X 25 and then add the two together [showing 86 X 86 and 47 X 25].

Of course in basic calculations this student had no difficulties. Questions such as 7300 X (47 + 78) = , finding powers, expressing fractions such as $\frac{23}{81}$ as decimals or as percents, were all done in a straight-forward manner. In the ninth week he did $\frac{63 \text{ X } 27}{933 + 44(17 + 3.8) - 234}$ by showing

63 X 27 = STO 933 + 44 X (17 + 3.8) - 234 = EXC ÷ EXC =

and when asked to do the question another way did

(63 X 27) ÷ (933 - 44 X (17 + 3.8) - 234) = . The second way was just slightly more efficient than the first, and even the second could have been simplified as well.

In doing two questions, 75 + 38 X 4 - 16 X 7 = and (75 + 38) X 4 - 16 X 7 = , this student calculated correctly and efficiently and was able to explain why the answers would be different.

During the interview prior to Checkpoint Number 6 a number of questions were asked about the operation of the calculator. He easily identified the function of the x^2 key, the $\sqrt{x}$ key, the $\frac{1}{x}$ key and the y^x key. He was able to explain how to set the calculator so that 16 could be subtracted from other numbers without the necessity of feeding 16 every time (making 16 a constant of subtraction). He made several attempts at this before he got it right but it was his own solution.

In doing Checkpoint Number 6, this student demonstrated clearly his ability in the use of the calculator and in applying it efficiently in mathematical situations. It also points out some of his weak spots.

1. He had no trouble establishing the sequence: 747, 723.4, 699.8, 676.2, ____, ____, ____. He did 747 - 723.4 = - K 747 = and this sequence correctly establishes the difference of the first two numbers, 23.6, as the constant of subtraction. By then feeding in 747 = after K and then following the last "=" sign with more "=" signs, the calculator will begin with 747 and in turn subtract 23.6 for as many times as one pushes "=". The one inefficiency in this sequence and in the next one demonstrated by this student was the failure to use memory, STO, to save 747. Had he done 747 STO - 723.4 = - K RCL = , he would not have needed to reenter 747. The second one had more complex numbers and there the use of STO would be a significant saving. However, the researcher considers the use of memory in this way a sophistication that would take constant use of a calculator in a wide variety of situations over a period of at least a school year.

2. In question 3. a) when required to divide 13.5 billion km by 300 000 km/s this student correctly used 13.5 EE↓ 9 ÷ 300 000 = . However, he did what almost all students did in this question and that is state an incorrect unit (km/s instead of s).

It would have been most efficient to realize that

13.5 ÷ 3 = 4.5 and then adjust the location of the decimal by noting that one billion is to be divided by one hundred thousand leaving ten thousand and so the result would be 45 000 s. However, this student did not attempt this or even comment on the possibility. No student realized that this was a method of attack.

3. In doing $18\frac{16}{23} = \frac{\quad}{23}$ and then using this result to change to a new denominator, $\frac{\quad}{23} = \frac{\quad}{322}$, this student easily did 18 X 23 + 16 = [430] and then 322 ÷ 23 = X 430 = . The one inefficiency was that he cleared 430 from the calculator and so had to reenter it but again this requires a high degree of ability in the use of the calculator. In doing 322 ÷ 23 = X 430 = he knew (when asked) that the first "=" is not necessary.

4. In reducing $\frac{22}{133}$, question number 5, this student showed some difficulty with prime numbers and reducing fractions. However, he recovered quickly under guidance.

Excerpt 15 (M-115):

R: So can $\frac{22}{133}$ be reduced?

S: Well, 3 doesn't go into 22 and 2 doesn't go into 133 since it's not an even number. 4......no, because......

R: Wait, 22 is what?

S: 2 X 11

R: So?

S: We checked 2 and 11.

R: So?

S: I guess you can't reduce them.

5. In question number 6, $24\frac{4}{7} \times 36\frac{14}{15}$, to be answered in common fraction form, this student began by wanting to multiply 27 by 36. In discussion we reached the conclusion that this question does not parallel addition where it would be possible to use such an approach.

Also in this question at one point (when he had $\frac{193}{7} \times \frac{554}{15}$) he wanted to multiply 7 by 15 with the intention, he said, of finding the common denominator. However, he caught himself and said he didn't need it.

6. In doing number 7, $18\frac{7}{9} - 12\frac{10}{11}$ he did $\underline{7 \div 9 + 18 = - (10 \div 11 + 12) =}$. After 11 he wanted to put an "=" sign (something he didn't want to do after the 9), but the researcher stopped him. The student himself then realized that if he did that, all operations to that point would be completed.

7. This student did questions 8, 9, and 10 with no trouble. He seems to experience, however, some difficulty with basic facts; this will be explored later. In question 10 a value of $31 025 was given as the cost of one hamburger at 85¢ a day for one year of 365 days. This is numerically correct but the actual cost should

be $310.25.

Excerpt 16 (M-115):

S: [Thinking aloud.] 85¢...365 days...[pause]... that's right. No, it isn't....hold on [did 365 X 85 on his calculator]. No, that's wrong.

R: Why?

S: It's only 310 dollars and a quarter.

R: Right.

S: That's [pointing to his display] 31 thousand and 25 cents.

8. In 11. b) where students were asked to calculate the answer for 49 + 85 X [ ] = 703.5, this student began by 703.5 - 49 ÷ and then:

Excerpt 17 (M-115):

S: Hold on, I don't want division [he is correct since he must evaluate 703.5 - 49].

R: Well, you can always divide by something and not change the answer. [Student then did 1 = following what he had already done, 703.5 ÷].

S: Now I can go to the division.

The researcher noted that he was the first student (of ten interviewed to this point, and as it turned out, the only one of the sixteen) to be able to solve this question. It appears that students typically did not see this as an equation. It also shows that this student solves an equation by a format which seems to be discouraged in school, $\frac{703.5 - 49}{85}$.

In question 9 involving estimation this student displayed some good mathematical thinking. Students were required to estimate 67.73 X 38.5.

Excerpt 18 (M-115):

[After establishing that it is about 70 X 40 or about 2800.]

S: 2800....well actually I'd say 2700 because they're both a bit high. [He was the only student who had even suggested this approach in estimating.]

9. In doing $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} =$ this student was going to do each fraction mentally and add 1 + 0.5 + 0.33, ... but the researcher suggested that this might be difficult when you get to fractions that are not obvious nor easy. He then did $\underline{1 + 1 \div 2 = + 1 \div 3 = +}$ and so forth to the end of the series. When questioned about the reason for the "=" signs he said that he just liked to see the intermediate results. However, the researcher told him that pressing the next "+" sign has the identical effect. When the student got to $\underline{1 \div 4 +}$ the researcher asked if there was another way (perhaps an easier one) by which he could continue.

Excerpt 19 (M-115):

S: Just do them in my head.

R: No, there's a way of using the calculator; we reviewed that earlier today. [No reply.] Well, finish the question as you were doing and write your answer. [He did.] Now start again and enter 1 + . Now what can you do to get 1 over 2. [No reply so instructor pressed 2 $\frac{1}{x}$.]

S: Why would I want to do that?

R: Because that key does exactly that. Whatever you have in the calculator it will do 1 over that number.

S: Oh, that's right. [He then finished the question in this way.]

10. The last question of Checkpoint Number 6 called for calculation of $^{-}64 + 198 \times {}^{-}32 - {}^{-}8344 \div 56 =$. This student ran into some problems because he wanted to apply his rules for integers and do the addition first. All other students proceeded to enter the question exactly as shown and of course with a calculator programmed to be algebraically correct this is a straight-forward question.

Excerpt 20 (M-115):

[Student began with 64 - 198.]

R: But that will be negative.

S: Isn't 64 - 198 the same thing as the other [$^{-}64 + 198$]?

R: Besides, don't you want to multiply these [198 X $^-$32]? [No reply.] The calculator will multiply first if you enter numbers in the order shown.

S: Yes.

R: But you don't want 64 minus that number [198 X $^-$32]. [Researcher wrote 64 - 198 X $^-$32.] You don't want to do this, do you?

S: It's the same thing, though. [Further discussion clarified this concept.]

A general observation about this student is that although he usually displayed good calculator usage and excellent knowledge of mathematics, he had trouble with some basic mathematical facts.

Excerpt 21 (M-115):

S: Because it would go 6 X 3 X 3 and then add 4 and then add the other 4.

R: What would it give you then?

S: That's 18 X 3, that's 30, 20, ... uh, ..., 50 some, 54.

Excerpt 22 (M-115):

[After completing $18\frac{7}{9}$ - $12\frac{10}{11}$ and getting 5.868 686 9].

R: Could that be correct?

S: Because 18 minus 12 is .... uh .... [long pause] 6.

Excerpt 23 (M-115):

S: That's about 3500.

R: No, it isn't. What are you doing?

S: 70 X 40.

R: So it isn't 35.

S: Why not?

R: What is 7 times 4?

S: Oh, 28; I was thinking of 7 times 5.

Excerpt 24 (M-115):

[Discussion on addition or subtraction of integers].

R: Is -5 + 7 the same as 5 - 7?

S: Yes.

R: What is -5 + 7?

S: 3

R: No.

S: Uh.... that gives you....uh....12.

R: No.

S: Yes, it does.

R: No, it doesn't.

S: It gives you negative 12, sorry!

R: No, positive 2 and the next one [5 - 7] gives you negative 2.

S: Oh, yes, that's right.

R: So these aren't the same.

S: Right. It's the other way around it's the same [7 - 5 same as -5 +7].

In reply to specific questions at the end of the project this student indicated:

1. He liked novel tasks on the calculator; for example, the "Talking" Calculator questions.

2. He found no parts particularly easy relative to others on the calculator.

3. He found complex fractions [which we did from time to time] difficult at first. "I had a bit of trouble with that at the beginning but then I found out how to do them."

4. He didn't use the calculator for many parts in mathematics class periods, "... because most of them were like 5 + 7 and stuff like that. It's mostly how to do it and not real tough numbers so I didn't use it for that."

5. He used the calculator when given equations had more difficult members.

6. He favors allowing use of calculators in school but not for tests and examinations.

7. He states that the calculator helps but does not show students how to do questions.

In the six-weeks follow-up (see Appendix I), this student did almost all questions perfectly. The only weakness was shown in question 9 where students were asked to convert each fraction $\frac{1}{17}$, $\frac{1}{45}$, and $\frac{1}{625}$ to a decimal. The most effective way would have been to use the $\frac{1}{x}$ key as in $\underline{17\frac{1}{x}}$. He did these as three division questions as $\underline{1 \div 17 =}$. For whatever reasons this student's retention of some of the procedures learned was excellent.

From the behaviors given it can be concluded that this student

-- is superior in using the calculator but still needs opportunities for additional practice and discussion about general calculator techniques.

-- is aware of the function of special keys of this calculator.

-- is excellent in estimating results.

-- excels in determining (and executing) calculations mentally when such is appropriate, but some basic facts seem to present difficulties unusual for a high-achiever.

-- analyzes questions very well before calculating.

-- is reluctant to write steps but rather proceeds directly to the calculation.

-- operates very well when numbers are large; is aware of how to use the calculator properly in handling such numbers.

-- can use the calculator for doing rational number algorithms and obtaining the answer in common fraction or decimal form; if the common fraction notation is required, more practice opportunities would be needed.

-- generally considers his problems so that often he realizes his own errors at the time that they occur.

-- is strong in mathematics but at times weak in some basic facts of arithmetic.

-- retains his knowledge extremely well about the use of the calculator. This was demonstrated in the six weeks follow-up.

-- has a high regard for calculators and a keen interest in using them.

In Q15 a number of characteristics are given for good calculator users. This student is very competent in a number of these; he estimates well, he is thoroughly familiar with the functions of the keys as covered in this project, he uses mental calculations when such are appropriate and does not rely on the calculator for these, he is confident in operating with large numbers, he is usually aware of the total problem and often realizes his own mistakes. On the other hand this student was weak in writing steps before doing calculations although he excelled in analyzing problems and determining what had to be done. He also needs more practice for many algorithms to become competent in all of these.

Case Study Number Two: M-118

This student is a male whose age was 13-7 at the start of the project. His IQ is recorded as 98 Verbal and 112 Non-verbal on the Canadian Lorge Thorndike Test. He entered grade 8 mathematics with a grade of 45 percent at the end of grade 7. He is a pleasant person, friendly, easy to talk to, and was interested in spending time outside of class in discussing the project or other matters of interest to him. He was keen about using the calculator and commented several times about how he enjoyed doing this work. Although he had had contact with calculators prior to the start of this project, it was only on a casual basis and like most students he had not worked with them in any regular or intense fashion.

In operating the calculator this student used his right hand (he was right handed) and frequently shifted his pen from hand to hand as he went from calculation to writing.

Generally he worked diligently on worksheets and assignments in class but occasionally had to be reminded to return to the assigned work. Indications are that he did not do much if any homework on calculator handout sheets as many were not done or only partially completed. Students were encouraged and urged to do the worksheets as close to completion as possible but no pressure was used.

Complex fractions were used at many points in the project since they tested students' ability to analyze a series of computations and organize their response. This set of questions and some series were not completed by this student. He also neglected to complete a page in which practice on using the constant (K) key was given and he omitted completely two pages on estimation practice. He had done most of the pages with number patterns but completely omitted pages two to six on "Working with Fractions" and two pages on "Some Further Patterns". This omission does not mean that he had no practice on these topics; it means he neglected to record answers for these questions completed in class, and he did not do additional ones in class when time was provided.

In general this student was not very strong in mathematics and not very strong in calculator usage. He easily learned some of the basics of calculator usage, but he had difficulty with more complex features. In the fourth week this student could not understand how $\frac{38 \text{ X } 14}{16 \text{ X } 4}$ = could be accomplished by the KSS $\underline{38 \text{ X } 14 = \text{STO } 16 \text{ X } 4 = \text{EXC} \div \text{EXC} =}$. He didn't know how to approach a problem such as 643 + 929 + [ ] = 2617. In the fifth week for $\frac{75 \text{ X } 47 - 63 \text{ X } 12}{16 \text{ X } 14 - 12 \text{ X } 8}$ this student produced $\underline{75 \text{ X } 47) - 63 \text{ X } 12 = \text{STO } 16 \text{ X } 14) - 12 \text{ X } 8 \div \text{RCL} =}$

and got 223.965 33, which is incorrect. To do his sequence correctly he would have needed "=" after 8 and then EXC ÷ RCL = . What he accomplished was $16 \text{ X } 14 - 12 \text{ X } \frac{8}{75 \text{ X } 47 - 63 \text{ X } 12}$. A basic error he made (in addition to neglecting to put "=") was to invert the complex fraction. In a follow-up lesson to this the above question was corrected and discussed. During the follow-up lesson students were given a similar one, $\frac{75 \text{ X } 47 - 63 \text{ X } 12}{16 \text{ X } 14 - 3 \text{ X } (38 - 47)}$ and this student correctly produced the following KSS:

75 X 47 - 63 X 12 = STO 16 X 14 - 3 X (38 + 47 = EXC ÷ RCL = .

In the seventh week this student still had a difficult time distinguishing between a KSS and a mathematical sequence. For $(-2)^2$ he showed -2^2 for the KSS rather than $2 \text{ +/- } x^2$. However in this week he did the test on integers and got two errors (one was a sign error) in the first thirty questions. He correctly did all the six equations but omitted four questions calling for evaluation of expressions (example: $n^2 - {}^{-}3 =$ ____ when n = -5).

In an interview in the seventh week he

--did 9 X 8 mentally.

--did 4 + 4 + 4 + 4 + 4 as $4 \; y^x \; 5 =$ but when asked to read the question realized his misinterpretation and corrected it to 4 X 5 = ; that is, he used the calculator for this.

--did 768 + 768 + 768 + 768 = as 768 X 4 = .

--did 8746 ÷ 100 on the calculator and after that he said he knew an easier way of doing it by counting two places and placing the decimal point.

--did 2 476 345 X 1000 by attaching three zeros to the original number.

--did 18 + 7 mentally.

--did 247 + 100 mentally.

--did 97 X 97 as $97\ x^2$ but wasn't satisfied so repeated as 97 X 97 = and noted the answer was the same.

In this same interview he had trouble with (86 X 86 + 47) X 25 + 17 = . If this question is entered exactly as shown the correct answer will be produced by the TI-30 calculator. This student's problem arose in attempting to rearrange the parts, assuming the answer would not change.

Excerpt 25 (M-118):

S: Do I have to keep it in brackets?

R: That's plus 47.

S: I know. Do I have to keep in brackets or can I go outside the brackets and then....

R: Do whatever you need to do to complete the question, but what does the question tell you to do?

S: 86 X 86 + 47, bracket, times 25 and plus 17. I was going to go, like 86 X 86 X 25 and then I was going to plus 47 and then 17.

This student is not sure about the necessity of parentheses. In the same question, (86 X 86 + 47) X 25 + 17, he could not explain the difference between the question as stated and a similar one without parentheses. When this student did a complex question and stuck to the given order he was able to solve it correctly. In the eighth week he had no problem with $37^4 - (16 \times 14^5 - 8) + (8 \times 10^6) + 4000$ mainly because he adhered to the order given.

His limited knowledge of some aspects of arithmetic were inhibiting in the calculator project. One particular frustrating experience is illustrated in Appendix L showing the difficulty this student had with a simple concept such as perimeter.

This student had some confusion in the language related to division. He did $\underline{231 \div 396 =}$ but talked about this as 231 into 396. In estimation he was reasonably accurate. In one page of estimation questions students were required to pick the largest value of three given (example: 33 X 64, 88 X 24, 76 X 29) by using estimates. They found the largest by using the calculator after all twenty had been estimated. Of twenty such questions this student got fifteen correct.

At the end of the calculator project when functions of keys were being checked this student

--knew that the x^2 key squares the display.

--did not know what $\sqrt{x}$ key does even though he tried $3\sqrt{x}$.

--knew that x^2 will check out a square root but this was after the square root key function had been explained.

--did not know what $\frac{1}{x}$ key does.

--knew what y^x key does and could explain it correctly in terms of the full multiplication; that is, 5 y^x6 is 5 X 5 X 5 X 5 X 5 X 5.

--knew how to load the calculator for repeated subtraction using the constant (K) key.

When the questions of Checkpoint Number 6 were put to this student he had trouble with many of them.

1. Even though students had done series several times in the project, this student could not begin to solve 747, 723.4, 699.8, 676.2, ____, ____, ____. He needed much help in establishing that it is decreasing by subtraction, by 23.6, and that one would repeatedly subtract 23.6. He did not know how to proceed to subtract 23.6 repeatedly by using the constant key. Even after having done 1. a) on subtraction he could not proceed with 1. b), a division situation. He had to be guided in determining the constant divisor. During this question he did not know the correct use of STO, RCL, and EXC.

2. In dividing 13.5 billion by 300 000 he knew how to enter scientific notation. He did know that this question called for division. He wondered why the calculator stays in scientific mode even though the result is not too large for the capacity of the calculator. (The calculator stays in scientific mode once it has been set in that mode and will stay that way until it is switched off.)

In expressing this answer, 45 000 s, in minutes, this student's first suggestion was to multiply by 60. He also had trouble seeing that (45 000 ÷ 60) ÷ 60 is the same as 45 000 ÷ 3600; he thought it would be the same as 45 000 ÷ 120.

3. In doing $16\frac{3}{7} + 38\frac{14}{19}$ in fraction format he used the calculator efficiently and easily arrived at $54\frac{155}{133}$. He could see that this would be 55 plus a fraction but to get that fraction he did 155 - 133 = . He had no trouble in establishing that $\frac{22}{133}$ is in lowest terms.

4. For $18\frac{7}{9} - 12\frac{10}{11}$ to be done in decimal form, this student did 18 X 9 + 7 = ÷ 9 = to get the first in decimal form. He needed help in ensuring that he had all the proper steps. The more efficient way would be to do 18 + 7 ÷ 9 = , saving three key strokes. His KSS was 18 X 9 + 7 = ÷ 9 = STO 12 X 11 + 10 ÷ 11 = and then he needed further help.

Excerpt 26 (M-118):

R: How can you now complete the question so that the numbers will be subtracted in the correct order?

S: You recall and then you minus ... no! That'd erase that [number in display].

R: Right, so what do you do?

S: EXC.

R: Right. [Student was hesitant--did not seem to be secure in how to handle this.] And then? [Student did EXC - RCL = but kept questioning researcher as to whether that was the correct procedure.]

5. When faced with number 10, the problem of the cost of an 85¢ hamburger every day for 365 days, this student thought it couldn't be as high as $365 (answer given in the problem was $31 025). When he calculated 365 X 85 = he was going to concede that $31 025 was right but he looked at it again (he couldn't leave it since he saw it could not surpass $365) and finally realized where the difficulty was.

6. When this student was given $49 + 85X = 703.5$ to solve he did $85X + 49 + {}^{-}49 = 703.5 + {}^{-}49$ but was reluctant to leave the right side in that form; he wanted to calculate it immediately. Even after being directed to leave it he could not seem to cope with that format, calculated it and then wrote $X = \frac{654.5}{85}$ for the last calculation.

7. This student correctly did $\underline{1 + 1 \div 2 + 1 \div 3}$, and so forth, for the KSS for $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{7}$. He did not, however, know about the use of $\frac{1}{x}$ as a possibility even though at the beginning of this checkpoint the function of the $\frac{1}{x}$ key had been reviewed.

8. When the last question was reached, $^{-}64 + 198 \times {}^{-}32 - {}^{-}8344 \div 56 =$, this student (as did several others) wondered about the use of parentheses around $^{-}64 + 198$. The researcher reviewed that the multiplication is to come first and that is what the calculator is programmed to do.

In reply to specific questions near the end of the interview this student indicated:

1. He liked to use the calculator when he had many questions to do such as multiplication questions.

2. He tried using the calculator on fractions but got all kinds of decimals which he could not understand so he did not use it for those again.

3. He feels constant use of the calculator would make you forget, "... such things as how to do fractions."

4. The calculator gives a tremendous advantage of speed.

5. He would not allow calculators to be used for examinations, "... cause they should know something instead of just knowing how to use the calculator."

6. One disadvantage he saw for the calculator is that a weak battery may cause the calculator to malfunction (produce unusual results) but the student would not be aware of this.

In the six weeks follow-up this student made a large number of errors. Obviously his retention was poor. Probably in the interim he had not had much opportunity to practice the same calculator skills he had been taught:

--in doing 37 X 46 + 37 ÷ 16 he used (37 X 46 + 37) ÷ 16 = and so changed the question.

--for $\frac{65 \times 12 + 4}{16 - 7 \times 18 \div 25}$ he used (65 X 12) + 4 = STO (16 - 7) X 18 ÷ 25 = ÷ RCL = . In this he made the error in inserting parentheses around (16 - 7) and then in doing = ÷ RCL = at the end he interchanged the roles of numerator and denominator.

-- he did $\frac{1}{17}$, $\frac{1}{45}$, and $\frac{1}{625}$ by using 1 ÷ 17, and so forth, to change the decimals; again he forgot about the role of $\frac{1}{x}$.

--in $34\frac{7}{9} - 15\frac{10}{11}$ to be done in decimal form he used 7 ÷ 9 + 34 = which is efficient, although at the end of the project he had been doing this as 34 X 9 + 7 = ÷ 9 = , a less efficient way. However, he could not cope with the total problem; he recorded each result and then keyed in again to perform the subtraction.

From the description of his behavior it can be concluded that this student

--is probably an average calculator user with some strong points but several weaknesses (he could not recall the use of $\sqrt{x}$ nor of $\frac{1}{x}$ keys although these were used several times in the project.

--is weak in many mathematical concepts which is reflected in his inability to cope with several of the established situations in the project; this is reflected also in his performance in using the calculator.

--is reasonably good in estimating results.

--needs much more help and practice with complex calculations and with order of operations before he would handle them efficiently; he needs to realize the correct order and how this is related to how the calculator operates.

--needs more practice with situations using STO, RCL, and EXC keys before he could utilize the full potential of this calculator.

--is reasonably secure in doing rational numbers in decimal form but would need more practice in working with mixed numbers where answers are to be expressed in fraction form.

Case Study Number Three: F-120

This student is a female whose age was 15-0 at the beginning of the project. Her IQ was recorded as 75 Verbal and 81 Non-verbal on the Canadian Lorge Thorndike Test. Her mark at the end of grade 7 mathematics was 42 percent. She had repeated grade 7 and so was older than most of the students although two other students in the project had repeated a grade. This student was pleasant and easy to talk to. In class she contributed almost nothing voluntarily. She would reply when questioned (if she had a response), but would sit silently if she did not have a response. In personal interviews she was willing to participate and give answers when she could. She did not seem to be discouraged by her many failures although the researcher generally tried to maintain non-threatening circumstances.

This student did not as a rule carry her calculator to class. She offered no explanation for this except to say that there wasn't much for which she needed it. However, four of the six students in her class did normally bring their calculators and used them regularly. Not using her calculator could be the reason she had not progressed much in calculator ability. Infrequent use of the calculator could be a reflection of her inability with it and failure to appreciate its potential in the grade 8 mathematics curriculum.

She used her left hand to handle the calculator. When clearing the calculator she pressed the ON/C key several times even though twice is all that is necessary. This seemed to be a nervous reaction and was the only outward indication of possible frustration. Otherwise she appeared calm and undisturbed by the many incorrect responses.

In doing $\frac{75 \times 47 - 63 \times 12}{16 \times 14 - 12 \times 8}$ in the fifth week she did 75 X 47 - 63 X 2 = (16 X 14 - 12 X 8 = . Besides the error of entering 2 instead of 12, she got the answer only for the denominator. Failure to enter "÷" after the first "=" meant she really began a whole new question at that point. A similar question was given in the ninth week. This was $\frac{63 \times 27}{933 + 44(17 + 3.8) - 234}$ for which this student supplied 63 X 27 ÷ 933 + 44 X (17 + 3.8) - 234 = so she solved $\frac{63 \times 27}{933} + 44(17 + 3.8) - 234$. She had no answer for the question that asked for a second (different) solution to this computation. This complex a problem was too difficult for this student to handle.

In the test on integers in the seventh week this student got all of the first thirty correct. Of the next six equations she had only one wrong and for the last four evaluations of expressions she had only one correct. This was quite unusual since students generally stronger than her in mathematics did not do as well in the test. The results do point out that in straight-forward calculations a poor student can be successful when analysis of the problems is not required. All of the first thirty could be done by just entering exactly what was shown (for example: $216 + {}^{-}387 + {}^{-}138 + 216 + {}^{-}416$). As a further indication of direct calculations, in the ninth week this student solved 7300 X (47 + 78) by 7300 X (47 + 98 = and did this again without using parentheses or memory by doing 47 + 78 = X 7300 = . She could do the fifth power of six by $6\ y^{x}5 =$ but could give no other ways of getting this question.

This student shows deficiencies in simple questions. She did 9 X 8 mentally and wrote 72 but for 4 + 4 + 4 + 4 + 4 she did 8 + 8 + 4 mentally. For 768 + 768 + 768 + 768 she added these on the calculator. She used the calculator to do 8746 ÷ 100 and seemed to have some vague idea about using a rule to divide by 100. She could not multiply by 1000 without guidance from the researcher. Such questions as 18 + 7 and 247 + 100 she did on the calculator. For (86 X 96 + 47) X 25 + 17 she got the correct answer because she simply had to enter exactly what she saw. For this one she said that the parentheses were necessary but could not explain why leaving them out gives a different answer. Only after assistance from the researcher could she see that in 86 X 86 + 47 X 25 + 17 the calculator does three additions, 86 X 86 to 47 X 25 to 17. In

(4 + 6 X 3) X 3 + 4 she could not state why the parentheses were needed although she did say that they had to be included or the answer would be different.

When given a problem requiring the calculation of the perimeter of a lot and then the cost of fencing it at $13.95 a metre, this student could not cope. The researcher helped her at great length and set it up entirely so that she just had to calculate to find the cost. This was in the eighth week.

Also in the eighth week she handled $14^4 - 2^{12}$ easily using the y^x key. However, in trying to use the calculator to factor 9555 she was unable to proceed to divide by consecutive primes. She needed careful step-by-step guidance in order to do this. She had no problem converting $\frac{7}{8}$ or $\frac{9}{14}$ to a decimal but could not express the latter to the nearest thousandth. She could not provide any clues how to proceed in finding the missing number in $\frac{14}{23} = \frac{}{736}$.

In the final checkpoint this student indicated the following:

--The x^2 key is used to multiply a number by itself.

--She did not know what the $\sqrt{x}$ key will do.

--She did not know what the $\frac{1}{x}$ key will do and even after it was established that $9\underline{\frac{1}{x}}$ will yield the value for $\frac{1}{9}$ she said that in doing this the calculator does 1 X 9.

--She had no idea of what y^x key will do.

--She did know how to make 12 a constant of subtraction in the calculator.

In the final Checkpoint, Number 6, the following observations were made. The interview was long since the student needed continuous help.

1. In deciding on how series can be continued she needed help right through the whole question. She could not establish the constant of subtraction and then use it to continue the series for the required number of entries. In the second part, a series requiring a constant divisor, she was very frustrated and pressed the ON/C key a total of eleven consecutive times at one point. She could not do it and again needed much careful guidance.

2. She did not know what to do to find how long it takes radio waves travelling 300 000 km/s to reach a point in space 13.5 billion km away. She also did not know how to enter scientific notation.

3. She could not complete the following:

$18\frac{16}{23} = \frac{\quad}{23}$.

4. In doing $27\frac{4}{7} \times 36\frac{4}{15}$ in fraction form she got to $\frac{193}{7} \times \frac{554}{15}$ and then multiplied to get $\frac{106922}{105}$ but beyond this she needed careful guidance to end up with $1018\frac{32}{105}$.

5. To change $18\frac{7}{9}$ to decimal form, this student wanted to do 18 + 9 X 7. It required careful explaining to establish the correct algorithm to do this. In the subtraction question, $18\frac{7}{9} - 12\frac{10}{11}$, she did know that when 18.777 778 was in storage and 12.909 090 in display, the question could be completed by using EXC - RCL = . In the next similar question involving division she proceeded reasonably well after the previous discussion but she neglected to put in an "=" sign in a crucial place in changing a mixed number to a decimal and so it did not yield the correct answer ($4\frac{15}{22}$ to a decimal should be 4 X 22 + 15 = ÷ 22 = but she omitted the first "=" sign).

6. She agreed that 365 hamburgers at 85¢ could cost $31 025 because she tried 365 X 85 = and got 31 025.

7. She could not offer any suggestions about how to solve 49 + 85X = 703.5. The researcher led her through that procedure in small steps.

8. She got $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$ easily by $1 + 2 \frac{1}{x} + 3 \frac{1}{x} + ... + 7 \frac{1}{x} =$. Because of her frustration the researcher did not pursue this further to see if she could suggest other ways of doing this.

9. In problem 14, because she is able to enter sequential operations she had no difficulty in correctly calculating the answer for $^{-}64 + 198 \text{ X } {}^{-}32 - {}^{-}8344 \div 56$.

In reply to specific questions at the end of the project this student indicated the following:

1. She had no comment when asked what kinds of questions she found easy on the calculator.

2. She made no reply when asked why she didn't use her calculator often in mathematics classes.

3. She had no opinion about whether the mathematics taken so far in school was easier or not because of the availability of a calculator.

4. She would not favor use of a calculator in tests, "Well, if you use them in class then you wouldn't know what you're doing on your math test."

In the six weeks follow-up this student did the first four basic calculations. She was not able to do a complex fraction and she did 23 X 23 X 23 X 23 X 23 by feeding this in as shown. For multiplying (and dividing) a series of numbers by a constant she did this by feeding each separately (12 X 7 = , 12 X 38 = , 12 X 49 = , and so on). She did $\frac{1}{17}$, $\frac{1}{45}$, and $\frac{1}{625}$ by 1 ÷ 17 = and so on. She did get $\frac{15}{22} = \frac{\quad}{264}$ by doing 264 ÷ 22 = and then using the result, 12, in 15 X 12 = . She did not even attempt two rational numbers questions (answers to be given in decimal form). She squared a number when the square root was called for. She was not able to do two last questions dealing with computations involving large numbers.

From these observations the conclusion is that this student, low in mathematical ability, has low calculator competence. She can do well, basic calculations where the order of the question dictates the KSS to be used but whenever some mathematical knowledge is required in order to do certain questions she is hopelessly lost.

Her knowledge of the calculator extends only to the basic four functions. At times she seemed to be able to use the y^x key for powers but she could not use x^2, $\sqrt{x}$, nor $\frac{1}{x}$ correctly consistently.

Her estimating ability is poor. In one page of twenty problems she got eight out of the twenty.

When faced with simple problems this student relies on the calculator and rarely does questions mentally.

Comparison of the Three Students

In the cognitive dimensions of mathematical knowledge and calculator knowledge, student M-115 would be rated High, M-118 Average, and F-120 Low. In syntax knowledge M-115 and M-118 were Average but F-120 Low. In the affective dimension of persistence M-115 was High, M-118 and F-120 Low. In attitude to calculator tasks M-115 and M-118 were High and F-120 was Average.

CHAPTER VI

CONCLUSIONS AND FURTHER RESEARCH

The conclusions that one might reach in a study of this nature could be wide and varied. Following are some conclusions that are indicated by the many observations made in the class, in personal interviews with students and teachers, and in evaluating the many tests given throughout the project. The conclusions are grouped into those that relate to teachers, students, the curriculum, the parents, and general mathematical difficulties. This order parallels the order of the research questions as given in Chapter II.

Following the conclusions some interpretations of the researcher are given. These are followed by suggestions for further research studies.

Conclusions

Calculator Usage and the Teachers

Teachers adapt to the presence of calculators in the hands of some students in their classroom by permitting them to use calculators whenever the students feel they can be used, but not for tests. Teachers do not make other adjustments such as directing special questions or specific procedures to calculator users. Teachers do, however, see a great deal of potential for calculators in the future and recognize the inevitability of their presence in society. They feel the schools should be adjusting to their presence with specific programs. However, teachers feel that there are fundamentals that must be taught so that students can do these without the use of the calculator. One teacher feels that there is plenty of work in grade 7 and to a large extent in grade 8 where students must learn basic calculations and the calculator should not be used unless it can be utilized to support those learnings.

Calculator Usage and Students' Reactions

The conclusions given below relate to all the sections of Chapter II that deal with the effect of calculator usage on students. The sections include regular student learning, going beyond the standard curriculum, high-achievers, low-achievers, the general use of the calculator, student attitude and interest, and additional changes in students.

1. Grade 8 students including even those low in mathematical ability can learn to use a scientific calculator efficiently for basic mathematical calculations including the use of a power function. Only the students

of higher ability have enough understanding of the calculator to be able consistently to use $\frac{1}{x}$, x^2, $\sqrt{x}$, y^x keys. However, even the student of lowest ability is able to use some of these keys, especially y^x, after much practice. Basic calculations involving integers ($^-216$ - $^-119$) become trivial even for low-achieving students using a calculator. When complex computations are required even low-achieving students are very proficient provided no major mathematical decisions need to be made. A question like $^-64 + 198 \times {}^-32 - {}^-8344 \div 56$ is simple because the calculator is programmed to follow algebraic order of operations. The student needs only to feed in the numbers and operations as shown. However, $\frac{63 \times 12 - 140}{114 - 73 \times 2}$ is different because a student must realize that the numerator and denominator must in some way be closed and the division of these then performed. Therefore brackets or storage features of the calculator must be employed. The success of students in such cases is not high and usually low-achieving students omit something (for example a parenthesis) which then changes the problem and leads to error.

Different algorithms need to be learned and stressed. In doing $\frac{3064}{32 + 93}$ using a calculator a student must ensure that 3064 is divided by the sum. With a paper and pencil approach this is quite clear but in using a calculator, doing 3064 ÷ 32 + 93 = is not sufficient. Students need to learn this and to have practice in such calculations.

2. Students can learn how to do operations with complex fractions using the calculator. With much repetition the low-achieving students are able to handle these in fraction or decimal form with the decimal form being much easier. Low-achieving students need much more time to become proficient at these since they tend to forget some important requirements. For example in doing $75\frac{4}{7} \div 30\frac{2}{13}$ using 75 + 4 ÷ 7 = ÷ (30 + 2 ÷ 13 = , the first "=" sign is essential as is the left parenthesis. Some students make syntax errors. It makes little difference to the students whether the numbers are large or small when doing rational number algorithms. There was unanimous agreement by the sixteen students that working in decimal form is preferred to working in fractional form. The ability to interpret decimal forms is not as well developed as interpretation of fractional numbers.

3. Students need more help and practice with analyzing problems and establishing the calculations necessary to solve the problems. Discussions relating to the necessary steps are required to make students more efficient calculator users.

4. Participation in the calculator project raised grades in estimation tests and in one test this was significantly higher than it was for the non-calculator students.

5. Students do not go beyond requirements of the program but they are interested in topics not in the normal curriculum when these are introduced by the researcher. Only one student showed interest in using the calculator for projects associated with other subjects. All students do apply their calculator talents to other subjects such as science or business mathematics. High-achieving students are much more active in class discussions pertaining to topics that are not normally discussed in grade 8 mathematics.

6. The best calculator users were the best mathematics students. They had some idea of the magnitude of results to be expected and therefore were alert to reasonableness of answers. The ability to analyze problems, especially complex computations, made some students superior in calculator usage. Using shortened procedures such as applying mental calculations helped to speed up the work.

7. Attitudes to mathematics were not significantly altered for the duration of the project. Interest in the project and in calculator usage remained high throughout the project and even at the time of the

follow-up six weeks after the conclusion of the project.

8. Calculator students progressed as well as non-calculator students in knowledge of the basic facts of arithmetic.

9. Calculator students progressed as well as might be expected in mathematics concepts and problem solving. They added on the average 4.69 points and 4.38 points respectively to the Grade Equivalent.

10. In rational numbers the calculator students did significantly better than the non-calculator students. On the Rate, Ratio, and Percent tests the difference between the means achieved by the two groups on the posttest was not significant.

Calculator Usage and the Curriculum

1. Good calculator usage can not be developed in a few months. It takes time for students to assimilate the techniques and the algorithms just as it would take with any other learnings occurring in mathematics. Even though some fears are continually expressed about the probable harm that calculators may produce, the evidence here shows that good calculator usage will not develop in an incidental manner (see Q15). Therefore, formal calculator education is necessary and could become part of the mathematics curriculum. If calculators are introduced early and used throughout school, students will

likely develop their sophistication and it will remain with them. Again this can be compared to what occurs at this time with the mathematics learning that takes place in school.

2. Calculations dealing with rate, ratio, or percent are trivial. The difficulty occurs in analyzing the problem to set up the correct proportion. Students seem to perform well with problems such as conversions among common fractions, decimal fractions and percent but the difficulties occur when students must set up such problems. For example, how does one establish a proportion for finding the percent that A is of B? High-achieving students establish patterns and memorize the three formats; low achieving students do not and so always have trouble with these questions, usually getting them wrong. However, once a problem is set up as $\frac{A}{B} = \frac{X}{C}$ where X is to be found, average and high-achieving students find no difficulty; low-achieving students need some guidance but usually get these correct once they reach the calculation stage.

3. In solving equations students usually used the format taught in schools, utilizing inverses. However, the high-achieving students shortened this process and calculated directly. The low-achieving students had much difficulty with equations having several steps and using large numbers. The problem was not the calculations but

the format to be used in solving equations.

4. There is sufficient challenge in grade 8 mathematics even if students use the calculator but in terms of calculations the present requirements provide little room for calculator usage. More challenging programs would need to be introduced so that the calculator would become a useful teaching device leading students to higher levels of mathematics.

Calculator Usage and the Parents

Generally parents of students in the calculator project support the use of calculators and indicate that they should be used in the junior high school. They support instruction on calculator usage in schools. All parents state that there will be a decline in basic skills and that students will become dependent on their use. However, parents recognize that calculators are a reality in society and schools should help prepare students to use them.

Calculator Usage and Mathematical Difficulties

The major difficulties from using the calculator stemmed from the students' problems with mathematics. The poorest student had no trouble in calculating such a problem as (A + B + C + D) X E once the problem had been analyzed and set up to that extent.

A number of difficulties in mathematics have been identified which do not relate directly to the use or non-use of a calculator. These difficulties are the points that need attention. The calculator could help improve performance in mathematics by a high factor if the basic problems in mathematics could be alleviated.

Students need more involvement with decimals. With the calculator, going from $7\frac{3}{11}$ to 7.2727272 is easy using 7 + 3 ÷ 11 = but this is not the usual way as taught in school. It would likely be 7 X 11 + 3 = and this would be written as $\frac{80}{11}$ rather than expressed as a decimal.

Students should be given a better understanding of the equality or inequality of such quantities as the following:

$A - (B + C)$ or $(B + C) - A$

$A - (B + C)$ or $A - B - C$

$A - (B - C)$ or $A - B - C$

$A \div \frac{B}{C}$ or $A \div B \div C$

$\frac{A}{B \times C}$ or $A \div B \div C$

$A \div (B \times C)$ or $A \div B \div C$

$\frac{A}{B + C}$ or $A \div B + C$ or $A \div (B + C)$

Interpretations

The conclusions reported previously were intended to be supportable by collected data. These conclusions and the extensive day-to-day experiences of the researcher allow for a number of interpretive inferences.

First, there is much learning that can be accomplished through the use of the calculator. This use extends far beyond that of a single computational tool. Discussions in grade 8 mathematics classes could centre around the analysis required in problem solving, and on the development of efficient calculator algorithms. These could be compared with pencil and paper algorithms. Thus, the calculator should enhance rather than inhibit mathematics teaching and learning.

Because of the large amount of mathematics development that can occur from the use of the calculator, it could be introduced much earlier than in grade 8. Evidence from this project shows that efficient and advanced calculator ability does not occur in three months. An extended period of time is necessary rather than trying to achieve these goals in a single calculator unit. The use of the calculator, therefore, should extend over several grades.

Teachers could incorporate calculators into regular programs but they need to adjust the curriculum and consciously prepare lessons with the calculator in mind.

Teachers should become familiar with the potential of calculators, not just at a simple four-functions operations level, but at a more sophisticated level. This is possible by using a scientific calculator, encouraging its use in the classroom and devising lessons around the calculator.

There is remarkable advancement in technology pertaining to calculators and the more advanced mini-computers (see Time, February 20, 1978 and Noyce, pp. 62-69). This implies that different levels of the educational structure must provide for the education of people to be able to cope with the rapid developments in this field. The need for knowledge in the area of calculators and computers extends not only to teachers but to universities in the preparation of teachers. There are implications for professional educational organizations such as teachers' associations, school boards, and ministries of education.

As seen in the study knowledgeable, educated teachers are not enough to ensure that a proper calculator program will come into effect. Teachers need the help of an active, positive policy towards calculator use for proper effects to come to full fruition.

Suggestions for Further Studies

This study considered a wide range of aspects of the calculator presence in two grade 8 mathematics classes. As a result the conclusions are based not so much on statistical evidence but more on observational data obtained through a variety of sources.

Conclusions are reported earlier in this chapter. Those dealing with the students and curriculum are strong conclusions because of the techniques that were used in gathering this evidence. The techniques used in gathering data on parents' opinions are probably reliable but not as extensive as other procedures might have been.

However, the conclusions on effects on teachers are based on limited techniques. This is in part due to the passive use of the calculator in the classroom. Only eight (or six) students, not all, in a class were provided with a calculator. In part the conclusions are based on weak methodological studies. This is recognized and it is suggested that further studies could utilize stronger methods in the aspects dealing with teachers. More structured, careful instructions could be given to teachers even to the extent of holding in-service sessions helping them incorporate the calculator into the total program.

The following suggestions for further studies arise from the study and are due in part to the variety of data-gathering techniques:

1. If the existing units are considered important to the junior high school then they should be revised incorporating the calculator. This may mean drastic revisions and would lead to a reassessment of priorities and emphasis in the various topics now taught. This move would require a researcher working with a few teachers and allowing materials to be tested and revised on a continuous basis.

2. What effect does verbalizing problems have on the ability of students to solve problems? That is, if students (all using calculators) are required to tell someone how they will solve the problem before they actually do it, will they be better problem-solvers than those who simply are allowed to do them?

3. A controlled experiment could be performed on the unit on rational numbers where all students use a calculator but one group works on the usual algorithms in fraction form while the other does all work in decimal notation. Students would be tested on understanding, estimation and application.

4. Students' ability to solve equations using the calculator could be studied. If students are taught algorithms more applicable to the calculator would they

still be well versed in the role of inverses in the solution of equations?

5. How does the calculator affect the way in which students approach problems?

6. Units such as Rational Numbers or Rate, Ratio, Percent could be selected for intensive study to see what students can or can not do using the calculator. This could mean few students and full concentration on one topic only. That is, a study that would be narrower than the one described here.

7. How can the calculator be used to develop particular mathematical concepts such as: percent and the relation to whole numbers, prime numbers and their role in factoring and hence in reducing fractions, better comprehension of magnitude of decimal fractions?

8. An intensive study should be conducted on the use of calculators with integers to see how students' knowledge of the rules is affected. Also appropriate algorithms would need to be developed since these change when calculators are used. How should students do $^{-}8476 - {}^{-}789$ when they have a calculator?

9. A study should be made of appropriate algorithms to solve rate, ratio, or percent problems. For instance the three basic types of percent questions should be studied and calculator techniques developed and tested to solve these and see how well students cope with

these problems.

10. What kinds of exercises using the calculator will in fact improve students' abilities in estimating skills, in rounding, or in achieving reasonable answers?

11. No-one has yet satisfactorily answered the question whether the use of the calculator over some period of time such as a year or more does in fact improve students' ability to estimate, or to round off, or to give reasonable answers.

12. Does the calculator enable students to better understand the relation of fractions to decimals; when do fractions repeat, which fractions yield terminating decimals?

13. Can grade 8 students master work with powers beyond that which is presently required and can they develop a full understanding of powers and algorithms involving them?

14. To what extent does calculator knowledge confound mathematical knowledge? An example of this is in the method of entering a constant of division into the calculator. In $\underline{A \div K\ Y =}$, Y represents the constant by which A and subsequent results will be divided if the "=" sign continues to be pressed. K represents the constant key. This method of feeding in a constant breaks the usual pattern of what we normally think of as a division situation.

15. In solving problems how much should students be required to write in order that they be efficient in problem solving? What should be written and what does not need to be written to ensure that students will have full understanding of what they have done?

16. Does the requirement of having to state in writing the KSS, make students better calculator users?

17. Will students who do the division of fractions using the decimal form on a calculator have a better (or poorer) understanding of division of fractions than do students who use the standard pencil and paper algorithm employing invert-and-multiply rule?

18. Could the calculator be used to help students overcome the confusion between A ÷ 0 and 0 ÷ A?

19. Can the calculator be used to increase the comprehension of large numbers by grade 8 students?

20. To what extent does the calculator assist students in learning the field properties for the real numbers?

21. Could the calculator be used effectively to reinforce certain rules of arithmetic; for example, such rules as the multiplication or division by 10 or powers of 10?

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APPENDIX A

DEFINITIONS

DEFINITIONS

1. Calculator: refers to the hand-held (or mini-) calculators in general or where clearly it is used to refer to the calculator in this project then it means a scientific calculator (the TI-30) as provided by the researcher for the study and having the following features:
   - basic four functions
   - $\frac{1}{x}$, x^2, $\sqrt{x}$, Y^x, π, %
   - sin, cos, tan, log, ln, degree key for working in degrees, radians or grads (together with an inverse key)
   - parentheses (multiple capacity)
   - +/- key for changing sign of display
   - EE↓ key for entering scientific notation
   - storage (memory) key (STO), recall key (RCL) and a key for adding directly into memory (SUM)
   - a key for exchanging the display entry with the quantity in memory (EXC)
   - an inverse key (INV) that can be used in a variety of ways (to get arc functions, hyperbolic functions, anti-logarithms, radian-degree-grad conversions, $\sqrt[x]{Y}$, change display format from scientific to regular where such is appropriate)
   - a constant (K) to enable the operator to make a

constant number to be used in addition, subtraction, multiplication or division.

2. Display: refers to the numerals visible on the calculator.
3. Calculator group: refers to the eight students from each of two grade 8 mathematics classes, taught as one group by the researcher.
4. Calculator students: the sixteen students who form the two calculator groups under study in this project.
5. Non-calculator students: the students in the two classes from which the calculator groups were drawn.
6. A single line under a sequence of numbers and operations or functions means an actual key sequence, e.g. 6 + 7 X 3 = indicates the pressing of the keys 6, +, 7, X, 3 and = in this order, to correctly evaluate 6 + 7 X 3 = .
7. Check: when used in combination as Opinion Check refers to a form of assessment or data gathering. The term "test" is not appropriate or may be misleading or perhaps threatening and so is avoided in certain instances.
8. KSS: Key stroke sequence -- this refers to the actual order in which keys are punched.

APPENDIX B

SUMMARY OF FIRST LESSONS ON BASIC OPERATIONS ON THE CALCULATOR

CALCULATOR ACTIVITIES -- GRADE 8

1. Observe the calculator -- do not turn it on.
   Note keys and their location.
   Function of each key is indicated above the key.
   Find the 10 numeral keys (0 to 9) and the decimal point. These are most important to us.
   Find the 4 keys for the basic operations -- +, -, X, ÷ -- also most important in our work.
   Find the = sign -- it is important because that is the way in which a question is closed (completed) and a final answer is obtained.
   Find the ON/C key, the OFF key -- C means clear (we shall discuss it in a short while).
   Other keys will be discussed as we come to them (of course the directions manual describes all keys and their uses).

2. Turn the calculator on by pressing the ON/C key.
   What do you see? (Small red '0' in the display).
   Leave it on just like that and do not press any keys for about 30 seconds. What happens? Explain (Travelling decimal point comes on -- display is maintained in the machine but is no longer visible). Pressing EXC key twice restores data which is in the machine, does not lose any calculation in progress and now allows operator to continue where he left off. This is a battery saving feature. If nothing should happen for about 10 minutes then calculator will turn itself off completely. However, as a regular point of procedure we should become accustomed to turning off the calculator whenever we are not going to use it for some time.

3. Enter 12345678. Watch your display (a good habit to begin to form) and notice how the digits jump into the display!
   With those numbers on display try to enter 9. What happens? Therefore what is the capacity of our calculator?
   Now press ON/C key. What happens?

4. Do these simple addition problems. Watch the display.

7 + 8 =

18 + 6 =

19 + 23 =

461 + 38 =

424 + 63 =

Notice that to enter any number such as 461 you press the digits in order just as you would normally write them.

Also the whole question is entered in order just as if you were writing it, always ending with the = sign. This sign is always the signal to the calculator to calculate the result and display the final result.

Let's try addition with some large numbers:

8 465 + 7 365

19 356 + 18 007 =

164 324 + 78 888 =

382 456 + 34 568 =

2 334 872 + 33 746 =

16 000 000 + 345 678 =

What can you say about doing this addition on the calculator as compared to some of the ones with smaller numbers done previously?

5. A look at the ON/C key more closely:
Enter 6 + 7 into calculator -- do not press = sign. Now press On/C once and then continue with 5 + 4 = . What is the result? Why? To understand it let's try this again with one difference:
Do 6 + 7 and now press ON/C twice. Now continue with 5 + 4 = . What do you get? Why?
One press of ON/C will clear only the number in the display but not your whole question. In the first case you still left 6 + in the calculator and only wiped out 7 sitting in the display -- when you pressed the ON/C once. When you continued with 5 + 4 = you in fact did 6 + 5 + 4 = . In the second case 2 presses of the ON/C key cleared everything from calculation so you then continued with a new question.

We must know how to use this key. Often we shall want to press this key once. Consider this example:

85 + 36 but now you notice that you really meant to press 46 so to correct this error, press ON/C once then continue with 46 = and you shall see the correct answer of 131.

Let's try this a couple of more times.

If a whole question is incorrect then two times on the ON/C key will clear everything and we start from the beginning.

Note: turning the calculator OFF and then on again with the ON/C key will also clear all calculations.

Note: it is not necessary to clear entry or total operation if the last key was the = sign. This key leaves the calculator clear and ready to go for the next question. This means that the old display will be replaced with your new entry.

6. Enter 98 765 432 and then square it. Use the x^2 key which means that this big number will be multiplied by itself.

   What is the meaning of this answer? (Scientific notation -- a brief description at this point).

   Let us continue squaring this big number three more times. This means we press x^2 three more times. What happens? (Error is displayed -- the capacity of the calculator has been exceeded). The 'Error' display will come up whenever we do something that the calculator is not programmed to handle or simply something which is mathematically false. Try $\underline{7 \div 0 =}$. What happens? Why?

   Note: whenever you see a single line under a key sequence which is written out then this tells you exactly which keys are pressed and in what order.

7. Let's try some subtraction questions: Enter the question and then obtain the answer.

65 - 37 =	86 - 9 =
457 - 363 =	222 - 195 =
8763 - 2437 =	95 438 - 23 458 =

   Are you watching your display to be certain the correct numbers are being entered?

8. Do the following:

   12 + 12 + 12 + 12 =
   867 + 867 + 867 + 867 + 867 =
   734 - 28 - 28 - 28 - 28 - 28 =

Remember that after an = sign is pressed, the next question may be entered without clearing previous display.

9. Try the following multiplication questions:

16 X 9 =   85 X 83 =   93 X 93 =
Can you do 93 X 93 in a different way?
673 X 37 =   746 X 372 =   4773 X 378 =
97 787 X 34 =   835 876 X 27 =   345 X 2354 =
56 X 100 =   6787 X 1000 =   873 452 345 X 10 =
67 X 34 X 24 X 8 =   687 X 27 X 15 X 4 =

Remember -- the calculator is a machine. It will do only what we make it do and only what it is built to do. It can not correct the mistakes made by the operator.

10. Do the following division examples:

17) 1462   89) 40584   375) 12375   444) 237984

2548 ÷ 98 =   9025 ÷ 95 =   1234321 ÷ 111 =

Do 16 ÷ 7 =   Why do you get such an answer?

Try 65 ÷ 4 =   Why is this decimal portion not as long?

Do the rest of these: Can you read the answer correctly using two corrected decimal digits but in fraction form -- for example, two and 13 one hundredths)

36 ÷ 16 =   48 ÷ 9 =   75 ÷ 11 =
867 ÷ 24 =   276 ÷ 64 =   9784 ÷ 635 =
87 345 ÷ 3125 =   88 ÷ 15 =   487 345 ÷ 4353 =
3 ÷ 4 =   15 ÷ 16 =   1 ÷ 7 =

Note: if we shall not be using the machine for a few minutes, we should switch it off even though it does have the built in switch. Not all machines have that and if we should be using a different machine a good habit to develop is to switch off when not in use.

11. Use of the K key. This key is used for entering a number to be used as a constant. (Why did the manufacturer select K to represent 'constant'?) This key is used when we wish to repeatedly add, subtract, multiply or divide the same number.

Key sequence for addition is m + K x - - - - etc. This sequence will keep adding whatever m is to the number represented by x. K is the key.

Let us try a simple sequence: remember that a single line under a series represents the actual key sequence that is to be used --

1 + K 10 = = = = Notice that we are getting the usual whole numbers beginning with 10.

Can you produce the series of whole numbers beginning with 150 up to 156.
(1 + K 150 = = = = = =)

Now make your calculator show:
Even numbers up to 20 (2 + K 2 = = etc. until 20 is displayed)
Odd numbers from 13 to 25 (2 + K 13 = = etc.)
8, 18, 28, 38, ... , 78 (10 + K 8 = = etc.)

Similar sequences can be used with the X, ÷ and - operations.

See if you can make your calculator show the following:
2, 4, 8, 16, ... , 8192 Write out all the numbers.
10, 30, 90, 270, ... , 196830
200, 175, 150, 125, ... , 0
167, 160, 153, 146, ... , 76
1024, 512, 256, 125, ... , 0
262 144, 65 536,16384, 4096, ... , 0.015625

Note: some people work the calculator with the hand which is not used for writing. In this way the 'writing' hand is free to record results or to do other required writing while working on mathematics.

12. Let's try some questions requiring more than one operation:

17 + 36 - 14 =
147 X 2 + 16 =
(17 + 25) X 18 =
(467 + 18) ÷ 53 =
(63 + 18 + 39 - 47) X 36 =

39 - 17 + 78 =
167 ÷ 14 - 4 =
What do parentheses mean? How do we key them into our calculator?

13. Paying attention to the order of operations:
What does 6 + 8 X 2 = mean? By the rules of mathematics we must do 8 X 2 first and then add the result to 6. This means that even though parentheses are not shown, we must do it as if they were around the 8 and the 2.
Our calculator is very special.

Try 6 + 8 X 2 = on your calculator. What did you get? Is that the correct response according to the rules of mathematics?

Now try the following, and remember that this calculator is programmed to follow the established rules of order of operations. This is not true of all calculators.

17 + 75 X 3 =
6785 + 420 X 50 =
353 + 46 X 27 ÷ 2 ÷ 3 - 200 =
375 + 34 X 12 =
674 - 14 X 26 =

Suppose our calculator did not have the correct order of operations built in.

Let us try the five previous questions but using our parentheses.

17 - 75 X 3 = now becomes 17 - (75 X 3) = .

Rewrite the others putting in the parentheses and then working them. Did you get the same results as previously?

This shows us that having this feature on the calculator saves us some time. We don't need to worry about where to insert parentheses.

However this feature means we must be cautious when doing problems. See if you can get the correct answer to the following problem:

We must take 27 and increase this by 45. Then it is necessary to multiply this result by 4.

Write the expression that will give us the correct result. Use your calculator to obtain the result.

Try another:

Begin with 180 and from it subtract 39. Now multiply the result by 17 and divide by 3.

Write the expression and calculate it. You should be able to see that at times we can make good use of the parentheses.

14. Our calculator also has a way in which we can store some information. The STO key can put a displayed number into storage (memory), the RCL key will bring the number out of storage (recall it) and put it into the display and the SUM key can be used to add the number in the display to the number in storage.

Let us try some questions in which we use the storage even though for examples chosen we have already seen that it is possible to do them directly or through the use of parentheses.

Suppose we wish to do the following:

$$\frac{17 \times 225}{16 - 7 + 36} =$$

Why can we not do 17 X 225 ÷ 16 - 7 + 36 = ? Or do you think that is correct?
What is the correct answer for the question? (85)
If we did it by the suggested way, what is obtained? (268.062 5)
Explain what the question is asking us to do and what the calculator did when we entered it as shown.

Let us now do this question using the storage (STO) and the recall (RCL) keys.
We shall first do 16 - 7 + 36 and put this into memory. Now we do 17 X 225, press the ÷ operation, recall the first result and get the calculator to give us the final answer by pressing the = sign key. The key sequence is
16 - 7 + 36 = STO 17 X 225 ÷ RCL = .

Can you explain the sequence? Did you notice that it was not necessary to press = after the 225? This is dangerous on this calculator since with some operations it would make a difference. Therefore we should always get into the habit of using the = sign.

Try another question of the same type:

$$\frac{695 \times 31 - 1421}{12 + 63 \times 8} =$$

Remember to do the denominator, put it into storage, do the numerator (do not forget the = sign -- it's important this time), then show ÷, recall the denominator and ask for the final result by pressing = .

APPENDIX C

WORKSHEETS GIVEN TO THE STUDENTS

CALCULATOR PRACTICE EXERCISES

1. Make your calculator display the following sequences. Write in only the two numbers which follow the first few given ones. You will know if you are right if you can reach the last number in the series.

a) 1, 2, 3, 4, ..., 20
b) 2, 4, 6, 8, ..., 30
c) 3, 6, 9, 12, ..., 45
d) 4, 8, 12, 16, ..., 60
e) 5, 10, 15, 20, ..., 75
f) 10, 20, 30, 40, ..., 150
g) 2, 6, 10, 14, ..., 46
h) 1, 4, 7, 10, ..., 49
i) 1, 11, 21, 31, ..., 151
j) 150, 155, 160, 165, ..., 220
k) 451, 459, 467, 475, ..., 531
l) 60, 107, 154, 201, ..., 577

2. Add: All the answers are to be found at the bottom of the page but not in the right order.

a)	b)	c)	d)	e)
793	2747	8674	25675	8
476	843	9733	4874	68
897	3767	6060	867	478
			77	9498

f)
8
98
998
9998

g) 6747 + 8748 + 48 =
h) 97·777 + 777 + 77 + 877 =
i) 83.5 + 67.33 + 783.77 =
j) 33.434 + 637.112 + 0.279 =

24 467
10 052
7 357

15 543.
670.825
99 508

31 493
934.207

2 166
11 102

3. Are the given answers correct? Draw a line through the wrong ones and write the correct one under.
Addition:

a)	b)	c)	d)	e)	f)
25	129	473	2794	4874	99
78	247	897	3842	2986	99
103	366	48	4768	79432	99
		1418	11304	97292	999
					1296

4. Add the following and record your answers:

a) 79	b) 247	c) 994	d) 6748	e) 1627	f) 7007
97	196	849	8784	897	6075
		78	8883	439	7060
			12948	7766	15000

Now add up all your answers and you should get 85 774. If you did not then find your error.

Check one: ____ I got the correct final sum

____ I made an error in ________________
(insert letters)

5. Subtract the following and record your answers. Correct answers are at the end of this question.

a) 89	b) 73	c) 175	d) 347	e) 6477
46	48	83	299	2003

f) 8007	g) 17 897	h) 85 700	i) 666 555
4968	4 623	47 993	499 388

92
48
4474
3039
25
13 274
37 707
43
167 167

6. This is another set of subtraction questions:

a) 7007	b) 678	c) 3478	d) 475.6
2078	79	999	38.9

e) 9774 - 8799 =
f) 1 767 969 - 999 497 =
g) 75.4 - 36.8
h) 97.77 - 43.765 =
i) 7.379 - 0.7777 =
j) 797.36 - 49.473 =
k) 9437.7 - 8379.49 =

54.005
1058.21
747.887
4929
599
768 472
6.6013
436.7
2479
975
38.6

7. Can you use the calculator and fill in the missing numbers: All are addition problems.

a) 38	b) 2□	c) 179	d) 6□3	e) 742
□	78	□3	□29	□
63	347	482	846	998
			1868	2107

$$\text{f)}\ \begin{array}{r} 600 \\ 20\square \\ 876 \\ \hline 1683 \end{array} \qquad \text{g)}\ \begin{array}{r} 63 \\ 4\square \\ \square 8 \\ \hline 168 \end{array} \qquad \text{h)}\ \begin{array}{r} 340 \\ 48\square \\ 9\square 9 \\ \hline 1729 \end{array} \qquad \text{i)}\ \begin{array}{r} 4076 \\ 8\square 7 \\ 2002 \\ \hline 14085 \end{array} \qquad \text{j)}\ \begin{array}{r} \square 342 \\ 4\square 63 \\ 22\square 7 \\ \hline 14382 \end{array}$$

8. Add or Subtract as required -- record answers:

a) 9767 + 2846 - 7993 =
b) 87.3 + 98.4 - 67 =
c) 38.4 + 39.56 + 79.379 - 120.939 =
d) 767 - 297 + 38 =
e) 38 - 33 + 47 + 53 - 60 =
f) 17.7 - 6.63 + 27.4 + 12.33 - 6.37 =
g) 7647 + 3333 - 2483 - 2244 + 1600 =
h) 616 - 225 - 207 + 78 + 8 - 33 =

9. Do the following. First circle your estimate for the closest answer and then calculate and write the exact answer in the blank:

a) 78 - 53 + 150 =	300	150	170	______
b) 867 + 347 - 463 =	750	950	670	______
c) 375 - 236 + 64 =	350	200	180	______
d) 6747 + 12300 - 8007 =	10 000	11 000	12 000	______
e) 38 - 20 + 67 - 15 =	80	60	70	______
f) 247 + 383 - 123 - 243 =	230	250	290	______
g) 333 + 463 - 700 - 3 =	90	190	60	______
h) 18470 - 12890 + 16000 - 4023 =	17 500	18 500	16 500	______

10. Figure out these sequences and insert the missing numbers:

a) 8764, 8616, 8468, 8320, ___, ___, ___, ___, 7580
b) 87.5, 81.2, 74.9, 68.6, ____, ____, ____, ____, 37.1
c) 1000, 798, 596, 394, ____, ____, ____, ____, -616
d) 1500, 1445, 1390, 1335, ____, ____, ____, ____, 1060
e) 21.1, 20, 18.9, 17.8, ___, ___, ___, ___, 12.3

11. Use multiplication to solve the following:

a) 16 + 16 + 16 + 16 + 16 =
b) 39 + 39 + 39 + 39 + 39 + 39 + 39 =
c) 473 + 473 + 473 + 473 + 473 + 473 =
d) 8167 + 8167 + 8167 + 8167 + 8167 + 8167 + 8167 =

12. Find the product for each of the following:

a) 67 X 5 =
b) 95 X 23 =
c) 847 X 3 =
d) 195 X 24 =
e) 24 X 195 =
f) 272 X 224 =
g) 224 X 272 =
h) 9 X 9 =
i) 64 X 64 =
j) 125 X 125 =
k) 2 X 2 X 2 =
l) 2 X 2 X 2 X 2 =
m) 5 X 5 X 5 X 5 X 5 =
n) 3 X 3 X 3 X 3 X 3 X 3 X 3 =
o) 18 X 16 X 15 =
p) 16 X 18 X 15 =
q) 15 X 18 X 16 =
r) 646 X 0 =
s) 0 X 646 =
t) 646 X 1 =
u) 1 X 646 =

13. Figure out these sequences and insert the missing numbers:

a) 1, 2, 4, 8, ___, ___, ___, ___, 256
b) 2, 6, 18, 54, ___, ___, ____, ___, 13 122
c) 8192, 4096, 2048, 1024, ___, ___, ___, ___, 32
d) 30 080, 15 040, 7520, 3760, ___, ___, ___,
___, 117.5
e) 0.1, 0.2, 0.4, 0.8, ___, ___, ___, ___, 25.6
f) 5, 0.5, 0.05, 0.005, ___, ___, ___, 0.000 000 5
g) 3, 24, 192, 1536, ___, ___, ___, ___, 50 331 648

14. Write the product for each of the following:

a) 47	b) 69	c) 347	d) 768	e) 8037
$\underline{13}$	$\underline{22}$	$\underline{28}$	$\underline{237}$	$\underline{6423}$

f) 6743 X 238 =
g) 24 756 X 47 =
h) 97.6 X 33.7 =
i) 999.9 X 10 =
j) 9.876 X 1000 =
k) 0.876 X 100 =
l) 0.876 X 9.876 =
m) 376.2 X 0.5 =
n) 7 647 124 X 0.75 =
o) 0.5 X 0.25 =
p) 38.125 X 0.125 =

1 604 834
1518
97.535 376
5 735 343
9999
51 621 651
9876
3289.12
987.6
4.765 625
9716
182 016
611
0.125
1 163 532
188.1

15. Find the quotient for each:

a) 75 ÷ 5 =
b) 874 ÷ 23 =
c) 2035 ÷ 37 =
d) 59 272 ÷ 478 =
e) 12 204 ÷ 18 =
f) 1536 ÷ 768 =
g) 16 482 ÷ 2747 =
h) 27 026 ÷ 8 =

i) $33\overline{)2541}$
j) $147\overline{)2352}$

124
678
6
55
77
3378.25
16
15
2
38

16. Further division questions:

a) $36\overline{)972}$
b) $18\overline{)2286}$
c) $134\overline{)12\ 998}$

d) $0.78\overline{)17.394}$
e) $0.07\overline{)1.1361}$
f) $34.8\overline{)2874.5766}$

g) 16 983 ÷ 37 =
h) 108 144 ÷ 18 =
i) 8121.7 ÷ 674 =
j) 771.034 ÷ 347 =
k) 338 800 ÷ 7.7 =
l) 469.365 ÷ 8.3 =
m) 78.787 ÷ 0.001 =
n) 0.00010816 ÷ 0.0104

6008
27
56.55
459
22.3
0.0104
12.05
97
76.047
16.23
127
4400
78 787
2.222

17. Write down the complete calculator display for the following:

a) 7847 ÷ 43 =
c) 93 464 ÷ 125 =
e) 367 475 ÷ 64 =
g) 774 217 ÷ 77 =

b) 6444 ÷ 44 =
d) 10 000 ÷ 55 =
f) 30 000 000 ÷ 512 =

747.712
146.454 55
10 054.766

58 593.75
5741.7969

182.488 37
181.818 18

18. Do the following as indicated:

a) 77 X 77 ÷ 77 =
c) 267 X 38 ÷ 47 =
e) 767 X 24 ÷ 16 =
g) 16 475 ÷ 256 X 16 =
i) 370 774 ÷ 3000 000 X 2 =
j) 7 747 893 X 2 - 625 =
k) 63 X 77 X 23 ÷ 16 X 2 ÷ 64 =
l) 999 X 999 X 20 ÷ 6735 =
m) 666 X 66 X 6 ÷ 111 ÷ 11 ÷ 1 =

b) 888 X 3 ÷ 11 =
d) 9743 ÷ 53 X 8 =
f) 8763 X 233 ÷ 25 ÷ 8 =
h) 240 693 ÷ 125 X 4 =

1029.687 5
1150.5
24 793.258
7702.176
10 208.895

2.471 826 7
215.872 34
242.181 82
1470.6415

216
77
2963.625 8
217.916 02

19. More sequences: try finding the pattern through division:

a) 17, 8.5, 4.25, 2.125, ____, ____, ____,
0.132 812 5

b) 65 536, 32 768, 16 384, 8192, ___, ___, ___, ___,
___, ___, ___, ___, ___, ___, ___, ___, 1

c) 59 049, 19 683, 6561, 2187, ___, ___, ___, ___,
___, 3

d) 33 480 783, 3 720 087, 413 343, 45 927, ___, ___,
___, 7

20. Still more sequences -- but these use all the basic operations. Can you fill in the blanks?

a) 1.6, 6, 10.4, 14.8, ___, ___, ___, ___, 36.8

b) 1, 8, 16, 24, ___, ___, ___, ___, 64

c) 1, 8, 64, 512, ___, ___, ___, ___, 16 777 216

d) 97.656 25, 39.062 5, 15.625, 6.25, ____, ___, ____, ____, ____, ____, 0.010 24

e) 7676, 7600, 7524, 7448, ____, ____, ____, ____, ____, ____, 6916

f) 530, 497, 464, 431, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, 35

g) 9, 31.5, 110.25, 385.875, ____, ____, ____, ____, ____, 709 340.75

h) 19 531 250, 3 906 250, 781 250, 156 250, ____, ____, ____, ____, ____, 10

21. Try these more difficult calculations:

a) $\frac{764 \times 13 \div 38}{9 \times 9 \times 7 \times 6} =$

b) $\frac{33 + 47 \times 18}{656 - 37 \div 2} =$

c) $\frac{788 + 134 \div 21}{77 + 88 \times 0.5} =$

d) $\frac{66 - 16 + 242}{743 \div 5 - 7} =$

e) $\frac{3346 - 45 \times 63}{247 \div 36 \div 5} =$

f) $\frac{222 \times 4 - 16}{7 \times 6 - 4 \div 16} =$

1.329 341 3
0.086 431 36
1.378 823 5
6.565 131 8
2.062 146 9
372.388 66

22. Let's see what happens to some fractions when we change them to decimals in the usual way using the calculator: We do this by dividing the numerator by the denominator. As you do these observe your results.

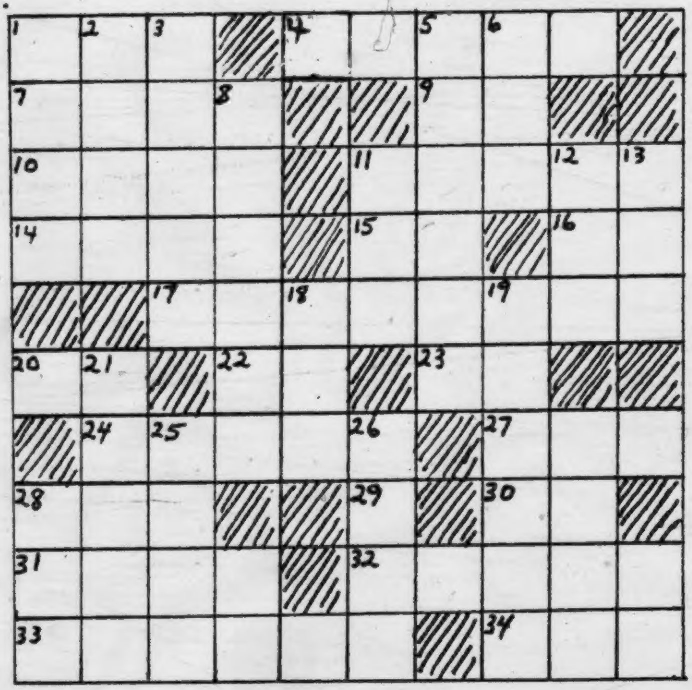
$\frac{1}{2} =$ $\frac{1}{3} =$ $\frac{1}{4} =$ $\frac{1}{5} =$

$\frac{1}{6}$ = $\frac{1}{7}$ = $\frac{1}{8}$ = $\frac{1}{9}$ =

$\frac{1}{11}$ = $\frac{1}{12}$ =

$\frac{2}{3}$ = $\frac{3}{4}$ = $\frac{2}{5}$ = $\frac{3}{5}$ = $\frac{4}{5}$ =

23.


1
2
3
4
5
6
7
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20
21
22
23
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31
32
33
34

This is a Cross-Number Puzzle.

Do the horizontal ones first and insert the numbers.

You will then be able to check by doing the vertical ones.

Parentheses are shown where necessary.

Horizontal

1. 267 + 80 =
4. 37 X 777 =
7. 2800 - 36 =
9. 77 X 2 - 107 =
10. 62 X 100 + 100 - 6 =
11. (75 X 75 + 586) X 10 + 1 =
14. 99 X 99 + 200 - 2 =
15. 2 X 2 X 2 X 2 X 2 X 2 - 2 =
16. 39 X 4 ÷ 3 - 11 =
17. 8888 X 8888 + 8 888 888 - 231 111 =
20. 9999 - 9922 =

22. 2 X 2 X 2 X 2 X 5 =
23. 1 000 000 - 999 990 =
24. (11 X 1000 + 111) X 2 X 2 =
27. (7 X 7 + 45) X 10 + 9 =
28. 1221 ÷ 11 =
29. 17 339 ÷ 2477 =
30. (3 X 47 X 10) ÷ 15 =
31. (111 X 11 - 110) X 2 =
32. 8888 X 8 - 1100 =
33. 330 000 + 3300 + 33 =
34. 256 X 25 ÷ 2 ÷ 2 ÷ 2 =

Vertical

1. 55 X 55 + (12 X 12) + 100 =
2. 10 000 - 5271 =
3. 76 X 1000 + 1000 - 3 =
4. (2222 ÷ 2 ÷ 11 - 100) X 2 =
5. 860 X 860 + 2641 =
6. 900 X 0.5 + 21 =
8. 670 X 670 + 29 X 29 + 43
11. 111 X 6 - 1 =
12. 12 X 12 - 2 =
13. 222 ÷ 2 - 1000 =
18. 2 X 2 X 2 X 3 X 5 X 5 + 4 =
19. 25 X 25 X 25 X 25 - 81 000 + 283 =
20. 777 777 ÷ 111 - 7000 =
25. 666 X 6 + 66 + 66 - 5 =
26. 9 X 5 X 100 + 200 + 73 =
28. 41 X 3 =

24. Decode this humorous message by calculating each letter value and then inserting the letters in the correct blanks:

A	909 - 900 + 90	P	79 + 37 + 65
B	2^6	W	779 - 550
O	4 X 6 X 5 X 2	J	444 ÷ 2 - 20
E	(80 + 175) ÷ 5	G	12221 ÷ 1111
I	73 + 27	E	282 ÷ 6
K	(75 - 44) X 10	U	3^6
D	998 + 889 - 1700	N	$8^2 + 9^2$
H	2^3 X 3^3	M	1728 ÷ 6 ÷ 2
B	11 + 99	U	25 X 25

S	23 + 15 + 24	Y	544 ÷ 4 ÷ 2
T	$2^7 \div 2^4$	F	64 + 35 + 15
R	63 + 147	S	$2^{10} - 10^3$
C	2 X 2 X 3 X 3 X 3	L	2^5 X 5 + 1
T	144 X 25 ÷ 30 - 40		

187 240   145 240 8   181 99 145 100 108

64 51 108 99 729 62 47   64 99 24 47

51 100 11 216 80   100 62   202 729 62 8

161 100 310 47   110 99 62 51   8 47 145'

100 114   68 240 625   229 51 210 47

144 100 62 24 100 145 11   80 229 240

114 100 145 11 47 210 24.

(Adapted from Iowa Council of Teachers of Mathematics--
original by Tom Lehrer)

Calculator Activities (Various Exercises)

Patterns

Each of the following is a pattern. It is not necessary to calculate every one and in fact at times it would be impossible to obtain the exact answer because it will go beyond the 8-digit display of the calculator. Once you see the pattern write in the remaining answers.

Part of the first one is done for you.

1. 1! = 1 This is read as one factorial.
2! = 2
3! = 3 X 2 X 1 = 6
4! = 4 X 3 X 2 X 1 = 24
5! = 5 X 4 X 3 X 2 X 1 = 120
6! = 6 X 5 X 4 X 3 X 2 X 1 = 120 X 6 = 720
7! = 7 X 6 X 5 X ... X 1 = =
8! =
9! =
Now do 11! by multiplying all the numbers:
11! =
What do you notice if you try anything beyond 11!? Try several and state what happens.

2. 3 X 37 =
6 X 37 =
9 X 37 =
Now guess the rest and write the answers without calculating
12 X 37 -
15 X 37 -
18 X 37 -
21 X 37 -
24 X 37 -
27 X 37 -
Can you explain why this pattern is like this once you know the first one?

3. 0 X 9 + 1 =
1 X 9 + 2 =
12 X 9 + 3 =
123 X 9 + 4 =
1 234 X 9 + 5 =
12 345 X 9 + 6 =
123 456 X 9 + 7 =
1 234 567 X 9 + 8 =
12 345 678 X 9 + 9 =

4. $1^2 =$
$11^2 =$
$111^2 =$
$1\ 111^2 =$
$11\ 111^2 =$
$111\ 111^2 =$
$1\ 111\ 111^2 =$
$11\ 111\ 111^2 =$
$111\ 111\ 111^2 =$

5. 1 X 0 =
11 X 1 =
111 X 11 =
1 111 X 111 =
11 111 X 1 111 =
111 111 X 11 111 =
1 111 111 X 111 111 =
11 111 111 X 1 111 111 =
111 111 111 X 11 111 111 =

6. 111 X 111 =
111 X 121 =
111 X 131 =
111 X 141 =
111 X 151 =
111 X 161 =
111 X 171 =

7. 6 X 7 =
66 X 67 =
666 X 667 =
6 666 X 6 667 =
66 666 X 66 667 =

8. 1 X 8 + 1 =
12 X 8 + 2 =
123 X 8 + 3 =
1 234 X 8 + 4 =
12 345 X 8 + 5 =
123 456 X 8 + 6 =
1 234 567 X 8 + 7 =
12 345 678 X 8 + 8 =
123 456 789 X 8 + 9 =

9. 8 X 88 =
8 X 888 =
8 X 8 888 =
8 X 88 888 =
8 X 888 888 =
8 X 8 888 888 =
8 X 88 888 883 =
8 X 888 888 888 =
8 X 8 888 888 888 =

10. 1 X 9 =
2 X 9 =
3 X 9 =
4 X 9 =
5 X 9 =
6 X 9 =
7 X 9 =
8 X 9 =
9 X 9 =
10 X 9 =
11 X 9 =
12 X 9 =
13 X 9 =
14 X 9 =
15 X 9 =
16 X 9 =
17 X 9 =
18 X 9 =
19 X 9 =

11. 1 X 99 =
2 X 99 =
3 X 99 =
4 X 99 =
5 X 99 =
6 X 99 =
7 X 99 =
8 X 99 =
9 X 99 =
10 X 99 =
11 X 99 =
12 X 99 =
13 X 99 =
14 X 99 =
15 X 99 =
16 X 99 =
17 X 99 =
18 X 99 =
19 X 99 =

12. 1 X 999 =
2 X 999 =
3 X 999 =
4 X 999 =
5 X 999 =
6 X 999 =
7 X 999 =
8 X 999 =
9 X 999 =
10 X 999 =
11 X 999 =
12 X 999 =
13 X 999 =
14 X 999 =
15 X 999 =
16 X 999 =
17 X 999 =
18 X 999 =
19 X 999 =

13. 1 X 9999 =
2 X 9999 =
3 X 9999 =
4 X 9999 =
5 X 9999 =
6 X 9999 =
7 X 9999 =
8 X 9999 =
9 X 9999 =
10 X 9999 =
11 X 9999 =
12 X 9999 =
13 X 9999 =
14 X 9999 =
15 X 9999 =
16 X 9999 =
17 X 9999 =
18 X 9999 =
19 X 9999 =

14. 9 X 9 =
99 X 9 =
999 X 9 =
9 999 X 9 =
99 999 X 9 =
999 999 X 9 =
9 999 999 X 9 =
99 999 999 X 9 =

15. 9 X 9 =
99 X 99 =
999 X 999 =
9 999 X 9 999 =
99 999 X 99 999 =
999 999 X 999 999 =

16. 9 X 9 + 7 =
9 X 98 + 6 =
9 X 987 + 5 =
9 X 9 876 + 4 =
9 X 98 765 + 3 =
9 X 987 654 + 2 =
9 X 9 876 543 + 1 =
9 X 98 765 432 + 0 =

17. $1 = 1 = 1 \times 1 = 1^2$
$1 + 3 = 4 = 2 \times 2 = 2^2$
$1 + 3 + 5 = 9 = 3 \times 3 = 3^2$
$1 + 3 + 5 + 7 = 16 = 4 \times 4 = 4^2$
$1 + 3 + 5 + 7 + 9 =$
$1 + 3 + 5 + 7 + 9 + 11 =$
$1 + 3 + 5 + 7 + 9 + 11 + 13 =$

You may have noticed that these are sequences of odd numbers.
Can you devise a method of calculating any number of odd numbers in sequence without going through the whole series?
Find the sum:

$1 + 3 + 5 + 7 + 9 + \ldots + 79 =$

If you have some difficulty, go back and see how many odd numbers there are in any series. Then see how you might figure that out.
Try several others such as:
Add up all the odd numbers less than 100
All odd numbers less than 200
All odd numbers less than 10 000

18. $1^3 = 1 = 1 \times 1$
$1^3 + 2^3 = 9 = 3 \times 3$
$1^3 + 2^3 + 3^3 = 36 = 6 \times 6$
$1^3 + 2^3 + 3^3 + 4^3 = 100 = 10 \times 10$
$1^3 + 2^3 + 3^3 = 4^3 + 5^3 = \qquad = \qquad X$
$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = \qquad = \qquad X$
$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = \qquad = \qquad X$

Can you predict the next ones without actually doing them?
$1^3 + 2^3 + 3^3 + \ldots + 8^3 = \qquad = \qquad X$
$1^3 + 2^3 + 3^3 + \ldots + 9^3 = \qquad = \qquad X$
Check your guess.

19.
1 X 9 - 1 =
21 X 9 - 1 =
321 X 9 - 1 =
4 321 X 9 - 1 =
54 321 X 9 - 1 =
654 321 X 9 - 1 =
7 654 321 X 9 - 1 =
87 654 321 X 9 - 1 =
987 654 321 X 9 - 1 =

20.
9 X 6 =
99 X 66 =
999 X 666 =
9 999 X 6 666 =
99 999 X 66 666 =
999 999 X 666 666 =

21. Can you guess how this pattern works?
Note that each number ends in a 5.
Could you do any up to 105 X 105 (or further) in your head?

5 X 5 =
15 X 15 =
25 X 25 =

35 X 35 =
45 X 45 =
55 X 55 =

65 X 65 =
75 X 75 =
85 X 85 =

95 X 95 =
105 X 105 =
115 X 115 =

125 X 125 =
135 X 135 =
145 X 145 =

When you catch on to the scheme do others by the short way:

155 X 155 =
165 X 165 =
175 X 175 =

22. Do each pair of calculations. How can you predict the second when you know the first? When you learn how then write the second answers using the first.

a	b
35 X 35 =	36 X 34 =
60 X 60 =	61 X 59 =
100 X 100 =	101 X 99 =
87 X 87 =	88 X 86 =
46 X 46 =	47 X 45 =
70 X 70 =	
95 X 95 =	

WORKING WITH FRACTIONS

1. Make these fractions equivalent:

a) $\frac{1}{2} = \frac{\quad}{18}$ b) $\frac{3}{4} = \frac{\quad}{28}$ c) $\frac{15}{17} = \frac{\quad}{357}$

d) $\frac{12}{13} = \frac{\quad}{195}$ e) $\frac{18}{23} = \frac{\quad}{1104}$ f) $12\frac{11}{47} = \frac{\quad}{47} = \frac{\quad}{1598}$

g) Write the key sequence for doing: $\frac{19}{37} = \frac{\quad}{3293}$

2. Decide which of the two fractions is the larger (or are they equal?). Put >, < or = between them. Write the numbers required for your decision on the blanks:

a) $\underline{\quad}\ \frac{1}{2} \quad \frac{3}{5}\ \underline{\quad}$ b) $\underline{\quad}\ \frac{3}{4} \quad \frac{19}{35}\ \underline{\quad}$ c) $\underline{\quad}\ \frac{7}{9} \quad \frac{11}{15}\ \underline{\quad}$

d) $\underline{\quad}\ \frac{12}{19} \quad \frac{119}{186}\ \underline{\quad}$ e) $\underline{\quad}\ \frac{91}{119} \quad \frac{13}{17}\ \underline{\quad}$ f) $\underline{\quad}\ \frac{7}{57} \quad \frac{8}{63}\ \underline{\quad}$

g) $\underline{\quad}\ \frac{16}{17} \quad \frac{259}{368}\ \underline{\quad}$

3. Factor these numbers. Use your calculator to assist you whenever you wish. Write the results in compact form as the example shows:

Example: $19800 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 11 = 2^3 \times 3^2 \times 5^2 \times 11$

a) 120 =
b) 462 =
c) 52 =
d) 663 =
e) 1380 =
f) 257 =
g) 1369 =
h) 1024 =
i) 105 468 =

4. Reduce the following fractions to their lowest terms:

a) $\frac{95}{125} =$

b) $\frac{210}{260} =$

c) $\frac{1275}{935} =$

d) $\frac{9409}{9797} =$

e) $\frac{13\ 778}{11\ 703} =$

f) $\frac{79\ 560}{1\ 646\ 892} =$

5. Do the operations as required with these fractions. The answers must be given in the usual fraction form:

a) $\frac{7}{10} + \frac{8}{9} =$

b) $8\frac{7}{11} + 37\frac{9}{13} =$

c) $\frac{2}{7} - \frac{1}{18} =$

d) $86\frac{7}{12} - 38\frac{14}{15} =$

e) $\frac{8}{17} \times \frac{24}{29} =$

f) $3\frac{2}{9} \times 4\frac{5}{7} =$

g) $97\frac{12}{17} \times 104\frac{75}{83} =$

h) $\frac{16}{33} \div \frac{6}{11} =$

i) $\frac{37}{47} \div \frac{9}{70} =$

j) $7\frac{9}{10} - 2\frac{4}{9} =$

k) $95\frac{17}{37} \div 12\frac{65}{73} =$

6. Take any whole number and do the following to it one after the other (remember that your calculator does correct order of operations so be careful):
multiply by 3, add 176, multiply by 2, multiply by 7½, subtract 2640, divide by 45.
What do you get? Does it always work? To answer this try a few more examples.

7. Repeat all the steps for number 6, but start with a number involving a fraction like $19\frac{7}{11}$ and write all the values in fractional form as you go along.

8. Do all the parts of number 5. again but this time work with the decimal form of the numbers.

What do you think about fractions done in fraction form as compared to the same questions done in decimal form when the calculator is used?

9. Canada became a nation on July 1, 1867; this is 110 years up to July 1, 1977. Calculate the following equivalences for 110 years. Start with the numbers of days using the fact that there were 27 leap years in that period.

a) How many days in the 110 years? ____________

b) How many weeks? ____________

c) How many hours? ____________

d) How many minutes? ____________

e) How many seconds? ____________

10. Can you put together combinations of just 2, 3, 5, and 7 in such a way that by multiplying and dividing only, the result will be 110? Two examples are given:
Example (1) $2 \times 5 \times 55 \div 5 = 110$
(2) $\frac{44}{22} \times \frac{25}{5} \times \frac{77}{7} = 110$

a)

b)

c)

11. Old Fashioned Problem:
Under the British system of measurement, a granary is found to be 12½ feet long, 8 5/6 feet wide and 10 7/8 feet high.
a) How many bushels of wheat can it hold if a cubic foot is 6¼ gallons and 8 gallons make a bushel?

b) How much does the wheat weigh in tons if a bushel weighs 60 lb?
Do both of these parts in fraction form first.
Next do the two parts again in decimal form. Think about the comparison of using the calculator in these two forms.

12. Problems on the calculation of π:
As you know π appears as the ratio of circumference of a circle to its diameter and is known to be $\pi = 3.14159265389793238 4626...$ and on and on never repeating in any pattern.
To calculate π, mathematicians have derived several formulas, some only approximate but others correct up to any point you wish to select for calculations.
Try the following and see what values of π you obtain up to the suggested point and compare to the value given above: Do all in decimal form.

a) Early Egyptian (about 1700 B.C.) formula for the area of a circle was $A = (d - \frac{1}{9}d)^2$ where d is diameter and d = 2r (i.e. twice the radius).

This formula gives $A = \frac{256}{81} r^2$ if we put 2r instead of d.

Using the r-formula calcualte the area of a circle of radius 25 cm.
Now calculate using $A = \pi r^2$ and π from the calculator. Compare the two areas.

b) Archimedes, a great ancient mathematician and scientist (287 - 212 B.C.), estimated π to be larger than 3 10/71 but less than 3 1/7. What is the difference in fraction form of these two numbers?
Using each of these find the largest value (in fraction form) for the area of a circle as well as the smallest (r=25 cm).

c) π can be obtained from the following:

$$\pi = 4 \times (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \ldots)$$

Find the exact fractional value that you get if you take the fractions up to 1/15 and then convert to decimal form.

d) Here is another value for π given by Abraham Sharp in 1717 (English mathematician, 1651 - 1742). He used it to get π to 72 decimal places. (Of course today π has been calculated to thousands of decimal places using computers).

$$\pi = 6 \times \frac{1}{3} \times \left(1 - \frac{1}{3 \times 3} + \frac{1}{3^2 \times 5} - \frac{1}{3^3 \times 7} + \frac{1}{3^4 \times 9} - \frac{1}{3^5 \times 11} + \ldots\right)$$

Write the series up to where you have 15 in the denominator. Again see if you can get the exact fraction up to that point. Then convert it to a decimal. Compare to the value for π given earlier.

e) Here is another English mathematician's value for π (John Wallis, 1655).

$$\pi = 4 \times \left(\frac{2}{3} \times \frac{4}{3} \times \frac{4}{5} \times \frac{6}{5} \times \frac{6}{7} \times \frac{8}{7} \times \frac{8}{9} \times \frac{10}{9} \times \frac{10}{11} \times \frac{12}{11} \times \frac{12}{13} \times \ldots\right)$$

Can you get the exact fraction from the above up to where it is written above? Note that your calculator goes into scientific notation so that would not be exact. Can you figure out how to get it exact? Now change it to decimal form. Compare to the given value of π.

13. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Using $\pi = \frac{22}{7}$ calculate the volume of a spherical gasoline storage tank 15 m in diameter (answer in fraction form). Do the problem again using the calculator value for π and get the answer in decimal form.

SOME FURTHER PATTERNS
(from a book by Judd)

1. 56 ÷ 11 =
   78 ÷ 11 =
   122 ÷ 11 =
   133 ÷ 11 =
   100 ÷ 11 =
   67 ÷ 11 =

   Can you add a few more to be divided by 11 to give you each of the patterns?

   Also try these and others of the same pattern:
   30 ÷ 11 =
   40 ÷ 11 =

2. Do 101 X 101 =
   101 X 101 X 101 =
   Guess 101 X 101 X 101 X 101 =
   and 101^5 =

   Can you prove that your guesses for the last two are exactly correct?

3. 101 X 11
   101 X 111 =
   101 X 1111 =
   101 X 11 111 =

   101 X 202 =
   101 X 2020 =
   101 X 20202 =
   101 X 2 020 202 =

   101 X 33 =
   101 X 333 =
   and so forth ----

4. Can you guess these even though your calculator can not give you the exact value?
   24 691 358 X 27 =
   X 45 =
   X 36 =
   X 54 =
   X 18 =
   X 108 =
   X 153 =
   X 123 =

5. 12 345 679 X 27 =
X 69 =
X 45 =
X 86 =
X 36 =
X 66 =
X 55 =
X 54 =

6. 1 X 9 + 2 =
12 X 9 + 3 =
123 X 9 + 4 =
1 234 X 9 + 5 =
etc. Where does this one end?

WHEN SHOULD I USE THE CALCULATOR?

1. Do the following without using your calculator:

Starting time: ________

4 + 8 + 2 = ________

20 - 10 - 4 = ________

12 X 100 = ________

2 X 3 X 2 = ________

40 ÷ 5 ÷ 2 = ________

1500 ÷ 3 = ________

$$\begin{array}{r} 45 \\ +23 \\ \hline \end{array} \qquad \begin{array}{r} 100 \\ -74 \\ \hline \end{array} \qquad \begin{array}{r} 45 \\ \text{X}5 \\ \hline \end{array} \qquad 4\overline{)120}$$

Finishing time: ________

Time needed without using the calculator: ________

2. Do the following using your calculator every time:

Starting time: ________

3 + 6 + 5 = ________

25 - 5 - 7 = ________

16 X 100 = ________

4 X 3 X 4 = ________

30 ÷ 3 ÷ 5 = ________

1800 ÷ 3 = ________

$$\begin{array}{r} 52 \\ +34 \\ \hline \end{array} \qquad \begin{array}{r} 100 \\ -67 \\ \hline \end{array} \qquad \begin{array}{r} 62 \\ \text{X}6 \\ \hline \end{array} \qquad 3\overline{)150}$$

Finishing time: ________

Time needed using the calculator: ________

USE YOUR CALCULATOR (And Your Head)

1. You are allowed to use only 1, 0, +, and = on your calculator.
   Make your calculator show your telephone number (actually you don't really need the = sign for this problem).
   What is the minimum number of entries you required? ____
   (An entry is any number entered in the display to be added to any other number).
   Can you think of a way to do it that will require an absolute minimum number of entries? What is that minimum? ________

2. Use your calculator efficiently to

   a) add 31.5 to each number

   76 ______ 347 ______ 9 ______ 47.9 ______

   300 ______

   b) add 489 to each number

   65 ______ 248 ______ 979 ______ 4965 ______

   348.65 ______

   c) subtract 87 from each number

   186 ______ 278 ______ 3789 ______

   56 345 ______ 478 346 ______

   d) multiply 1.08 by each number

   73 ______ 646 ______ 342 ______

   75.94 ______ 376.873 ______

   e) divide each number by 47

   799 ______ 31 161 ______ 47 ______

   119 911.15 ______ 3760.438 ______

f) divide each number by 35.8

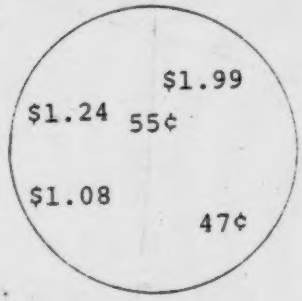
3043 ______ 27 745 ______ 3542.2 ______

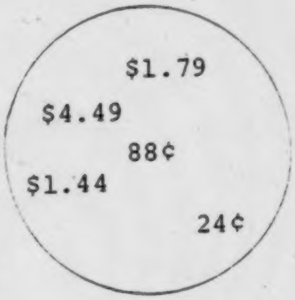
345.47 ______ 1281.64 ______ 35.8 ______

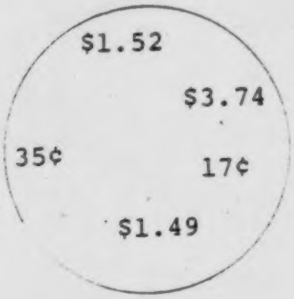
ESTIMATION

Each circle contains prices of a number of items purchased.

1. Estimate the cost of the prices in each circle and write your estimate in the proper space. Do all estimates first.
2. Calculate actual costs and enter those.
3. Calculate the difference between your estimate and the actual cost.
4. Can you give the percentage error (based on the actual cost)?


$1.99
$1.24
55¢
$1.08
47¢

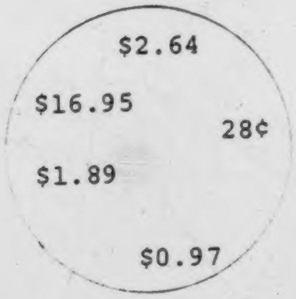

$1.79
$4.49
88¢
$1.44
24¢

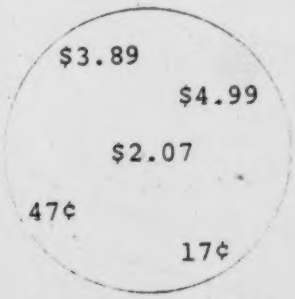

$1.52
$3.74
35¢
17¢
$1.49

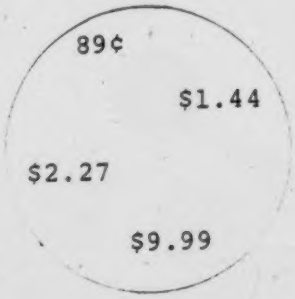
Estimate ________
Actual Cost ____
Difference _____
% error ________

Estimate ________
Actual Cost ____
Difference _____
% error ________

Estimate ________
Actual Cost ____
Difference _____
% error ________


$2.64
$16.95
28¢
$1.89
$0.97


$3.89
$4.99
$2.07
47¢
17¢


89¢
$1.44
$2.27
$9.99

Estimate ______	Estimate ______	Estimate ______
Actual Cost ___	Actual Cost ___	Actual Cost ___
Difference ____	Difference ____	Difference ____
% error _______	% error _______	% error _______

HOW'S YOUR GUESSTIMATION

Target Practise: In this game you start with the first number and try to hit the second number through multiplication. Each guess must be recorded in the spaces provided.
Try to hit the target in the least number of trials. If you need more trials than spaces provided, write extra ones beneath the ones given.

1. 43 ____, ____, ____, ____, ____, ____, ____, 387
2. 75 ____, ____, ____, ____, ____, ____, ____, 1350
3. 89 ____, ____, ____, ____, ____, ____, ____, 1246
4. 101 ____, ____, ____, ____, ____, ____, ____, 1111
5. 101 ____, ____, ____, ____, ____, ____, ____, 1515
6. 101 ____, ____, ____, ____, ____, ____, ____, 1818
7. 101 ____, ____, ____, ____, ____, ____, ____, 1919
8. 101 ____, ____, ____, ____, ____, ____, ____, 2929
9. 11 ____, ____, ____, ____, ____, ____, ____, 121
10. 11 ____, ____, ____, ____, ____, ____, ____, 1221
11. 11 ____, ____, ____, ____, ____, ____, ____, 12 221
12. 11 ____, ____, ____, ____, ____, ____, ____, 122 221
13. 11 ____, ____, ____, ____, ____, ____, ____, 1 222 221
14. 48 ____, ____, ____, ____, ____, ____, ____, 134.4
15. 27 ____, ____, ____, ____, ____, ____, ____, 472.5
16. 76 ____, ____, ____, ____, ____, ____, ____, 1425
17. 95 ____, ____, ____, ____, ____, ____, ____, 3576.75
18. 125 ____, ____, ____, ____, ____, ____, ____, 1110
19. 156 ____, ____, ____, ____, ____, ____, ____, 8221.2
20. 985 ____, ____, ____, ____, ____, ____, ____, 8732.5
21. 1000 ____, ____, ____, ____, ____, ____, ____, 49 600
22. 22 ____, ____, ____, ____, ____, ____, ____, 488 884
23. 16.5 ____, ____, ____, ____, ____, ____, ____, 14 850
24. 20 ____, ____, ____, ____, ____, ____, ____, 17 556
25. 968 ____, ____, ____, ____, ____, ____, ____, 10 551.2
26. 3748 ____, ____, ____, ____, ____, ____, ____, 71 212

MORE ESTIMATION

For each of the following guess your answer and then check on the calculator. Solving these as equations is not allowed.

Record each of your trial numbers to the right of the question.

Two spaces in one question require the same number.

1. 15 X [ ] + 16 = 55
2. 75 - 16 X [ ] = 31.8
3. 65 X [ ] = 936
4. 175 + 247 - [ ] = 38
5. [ ] X [ ] = 1122.25
6. [ ] + [ ] = 2437.2
7. 85 X [ ] = 6383.5
8. 175 + 25 X [ ] = 237.5
9. 28 + 28 + 28 + [ ] = 182
10. 7948 - [ ] ÷ [ ] = 7947
11. 175 ÷ [ ] = 70
12. 452 ÷ [ ] = 14.125
13. [ ] X [ ] + 450 = 7846
14. 75 ÷ 40 + [ ] = 4.475
15. 27 ÷ [ ] = 0.84375
16. 27 ÷ [ ] = 8.4375
17. [ ] - [ ] ÷ 17 = 0
18. 6548 + [ ] X [ ] = 7277
19. [ ] ÷ 25 = 75.8
20. [ ] ÷ 12 X 4 = 460

21. 175 + 2 X [ ] = 311

22. 6 X [ ] + 249 = 345

23. 38 + 75 X [ ] = 1350.5

24. [ ] + 38 X 48 = 1921

25. [ ] + 1143 ÷ 500 = 185.286

26. 75 ÷ [ ] X 53 = 159

POWERS

1. Complete these powers and see the differences:

   $(3^2)^5 =$

   $3^{25} =$

   $(3^5)^2 =$

   $3^{2^5} =$

   $3^{5^2} =$

   $32^5 =$

2. Start with 2. Double and get 4. Double again and get 8. Double once again and get 16.
   Guess the number of times required to reach 1000. ____
   Now repeat this on your calculator and record the number of times required to reach (or pass) 1000. ___

   Write this as a power notation: ______=______

3. Repeat the previous activity but starting with 3 and doubling every time.

   Guess the number of doubles required to reach 1000. __

   Repeat on the calculator and record the number of doubles. _________

   Can you write out the multiplications you performed?

   ____

4. The story is told of a man in ancient times who performed a special task for the emperor. In order to reward the man the emperor asked him what he would like.

   The man replied, "I do not want money. Grant me 1 kernel of wheat for the first square of the chess board, 2 kernels for the second square, 4 kernels for the third and so forth doubling the number of kernels for each square up to all of the 64 squares."

   The emperor was pleased to grant so simple a wish and set his men to fulfill the request.

   Calculate the number of kernels the man would be entitled to receive for the 64-th square.

   ____

   Can you calculate the total number of grains he should receive (approximately)? ________________

   Did the man strike a good bargain?

   I wonder how much that amount of grain would bring at $90 a tonne, which is 1000 kg.

THE "TALKING" CALCULATOR

For each of the following, do the calculation and then rotate the calculator and read the message:

1) An informal greeting: $(28 \times 6 + 74) \div 11 - 8 =$

2) An informal greeting to a female friend:

$$851.19 \times \left(\frac{2454 - 600}{100} - 0.54\right) \div 3 =$$

3) A more formal greeting to a male friend:

$$(675.981\ 46 \div 803 \div 2 - 0.000\ 91)0.184 + 0.000\ 06 + 808 =$$

4) All this calculator work may do this to your mind:

$$(10\ 101 \times 30 + 7777)1.05 - 39.35 + 53\ 600 =$$

5) What Amelia Earhart put on for take-off: (She was a famous woman pilot)

$$(25\ 678.75 \times 24) + (5 \times 1000 \times 1000) - 444\ 444 + 208\ 063 =$$

6) And this is what Amelia's father said:

$$(250.45 \times 200.3) + (3457.344 \div 6.4) =$$

7) On our trip we stopped here several times:

$$(33.75 \times 20.37 \div 0.9) - (106.203\ 1 \div 2) =$$

as well as here:

$$(58.609\ 04 \div 7 \times 25 - 206) \div 6 \div 10 + 1000 - 290 =$$

8) JAWS had a very large appetite. In fact he ate so much of so many different things he was called a:

$$((2 \times 10^6 + 547\ 747) \div 9 \times 30 - 416\ 888 + 316) \div 1000 =$$

9) There are too many of these in the roadways in the spring:

$$53 \times 1000 + 720 - 12 - 4 + 0.918 =$$

10) What the sleepy resident threw at the howling cat:

((440 + 44 + 4 + 3) X 6 + 6 + 0.973) X 9 X 2 =

11) What Snoopy gets a lot of in his doghouse as a result of his scraps with the cat next door:

$2^{15} + 2^{3} + 2^{6}$ X 3 X 109 =

7

PROBLEMS I

1) A person goes on a car trip from Edmonton to Saskatoon. From there this person continues on to Regina, then to Calgary and finally back to Edmonton. The distances for each part of this trip are (in order) 544 km, 263 km, 760 km and 288 km.

a) How many gallons of gasoline would be used if the car averages 31.6 km/gal?

b) How much does the gasoline cost at an average of 87.9 ¢/gal?

c) What is the actual travelling time if this person goes at an average of 90.4 km/h?

2) The hamburger problems:

a) Suppose that MacDonald's advertises that it has sold 18 billion hamburgers altogether in the United States and Canada. If each hamburger is assumed to be 4 centimetres thick, how high would a stack of 18 billion hamburgers reach (if we can picture them as forming one large stack)?
Give the answer in cm first and then in m and finally in km.

b) The distance around the world at the equator is about 41 000 km. How many times is the stack of hamburgers as much as the distance around the earth?

c) In the United States there are about 220 000 000 people and about 23 000 000 in Canada. If we assume that each of these people eats one hamburger a day, how long would it take to eat 18 billion hamburgers?

d) Each hamburger uses about 125 grams of beef. How many kilograms of beef are needed for 18 billion hamburgers?

e) How many tonnes of hamburger meat is needed for the 18 billion hamburgers?

f) Let us take the average cost of a hamburger as 65¢ (some cost less and some cost more). How much money would be spent for 18 billion hamburgers at this rate?

g) The government of Alberta just announced that during this next year it is going to cost 3.8 billion dollars for its expenses in running Alberta government business. How many times as large is the cost of the hamburgers as this Alberta budget?

3) Why chain letters requiring some gift are illegal: Suppose you started a chain letter and sent it to 6 people only and each of these is required to send it to 6 new people. Call the first sending (by you) round 1 and the next one round 2. If noone gets more than one chain letter in which round would you go past the total population of Canada (23 000 000)?

In which round would you pass the total world population (4 000 000 000)?

CALCULATOR ACTIVITIES (VARIOUS EXERCISES)

ESTIMATION

Each question has three calculations. Draw a ring around the one you estimate is the largest (do not calculate exact answers). Next use your calculator to get all three answers and then put a rectangle around the largest of a, b or c. Is the rectangle around the same one as the ring?

	a	b	c
1.	33 X 64	88 X 24	76 X 29
2.	97 + 64	127 + 48	95 + 90
3.	740 - 270	1800 - 1237	1176 - 444
4.	75 X 15	23 X 39	1510 - 720
5.	55 X 49	153 X 20	666 X 4
6.	747 + 262	33 X 27	44 X 24
7.	63 ÷ 37	19 ÷ 9	35 ÷ 25
8.	14 X 8 X 7	15 X 10 X 6	15 X 9 X 6
9.	540 ÷ 52	475 ÷ 23	98 ÷ 3
10.	1000 - 497	247 + 167	22 X 19
11.	710 ÷ 11	7035 ÷ 120	76 ÷ 2
12.	365 X 4	353 X 5	420 X 3.8
13.	897 + 245	2100 - 140	1850 ÷ 2
14.	6 X 16 + 20	28 X 3 + 5	15 X 6 - 3
15.	95 X 9 X 8	85 X 11 X 2	178 X 0.5 X 15
16.	$18\overline{)600}$	$202\overline{)1500}$	$12\overline{)800}$
17.	93 + 48	16 + 79 + 23	17 X 12

18. 2 X 2 X 2 X 36   3 X 3 X 47   4 X 4 X 60

19. 437 - 15 - 18   1563 - 1450 + 225   721 - 285 - 60

20. 36 X 36   37 X 35   41 X 32

APPENDIX D

SUMMARY OF CLASS LESSONS

SUMMARY OF CLASS LESSONS

Although procedures at each of the two schools varied somewhat only one summary is given. Where variations occur these are noted. Summaries are given for each of the twelve weeks (thirty-six forty-minute class periods in total).

Week 1 (Lessons 1, 2, 3)

Introduction to the use of the calculator and the functions of the keys.

"Error" display when overloaded or when an unacceptable operation is attempted ($a \div 0$)

Possible use of $\frac{1}{x}$ key in $\frac{36}{18 \times 4}$ = by $\underline{18 \times 4 = \frac{1}{x} \times 36 =}$

Gave first 4 pages of practice exercises

Week 2 (Lessons 4, 5, 6)

School A:
How to use the constant (K) key
Using the K key to do sequences (3, 15, 75, 375,

_____, _____, _____, 234 375.)

Use of y^x key to do powers (review)

Briefly--the use of tangent function for finding flagpole height

Did more examples of complex fractions and different ways to do them

$$\text{e.g.} \quad \frac{75 \times 16}{12 + 38}$$

Gave pages 5 and 6 of practice exercises

School B:
Use of y^x key
Tangent function--flagpole height (this arose in response to a question)
Interviewed students about 2/3 of the time

Week 3 (Lessons 7, 8, 9)

Timed test on very simple calculations--with calculator compared to time without the calculator

Telephone number problem--make your calculator show your telephone number by minimum number of moves and using only 0, 1, + (and possible =)

Constants of operations using the calculator

Hitting a target number from a given number--using multiplication and keeping the guesses to a minimum

Estimation of Cost--worksheet

Two ratio of costs problems--Macleans magazine (cost for two different lengths of time)--Pears soap cost (which is the better buy--large or small bar)

Gave pages 7, 8 and 9 of practice sheets

Guesstimation page--worked several examples as models

Reviewed calculation of percent or error (needed to do the worksheet where costs displayed in circles were to be estimated and calculated--Appendix C)

Reviewed sequences (15, 17.5, 20, 22.5, ____, ____, ____, ____, 35)

Reviewed use of reciprocal key (1/x)

How do you recognize some decimals to be repeating decimals from the display (3.2727273 is really $3.\overline{27}$ -- the 3 comes from the rounding done automatically by the calculator)

Function of the = key--it completes all calculations pending to that point--how to use it properly in such questions as $\frac{75 + 18}{12 \times 7}$ which can become 75 + 18 = ÷ (12 X 7 = . The first = sign is necessary to complete 75 + 18 so that the sum is divided by the product of 12 and 7. Omitting the = sign causes only 18 to be divided by the product and then the result added to 75. The right parenthesis is not needed as the last = sign will automatically complete all pending operations correctly.

Reviewed order of operations

Did some personal interviews.

Week 4 (Lessons 10, 11, 12)

Prepared for Group Test No. 1 with examples

Gave Group Test No. 1 (students had to give KSS and the answer)

Again did complex fractions and the various ways of doing them:

$$\frac{38 \times 14}{16 + 8}$$

Gave More Estimations worksheet-discussed briefly what is expected

Missed one class period--School A because of Teachers' Professional Development Day

--School B because the researcher had to leave early (students worked on their own with supervision provided by the school)

Week 5 (Lessons 13, 14, 15)

Gave one special problem for which KSS and answer were to be given:

$$\frac{75 \text{ X } 47 - 63 \text{ X } 12}{16 \text{ X } 14 - 12 \text{ X } 8} =$$

--discussed various ways of doing this

--also gave another variation of it for further practice (did this and discussed ways):

$$\frac{75 \text{ X } 47 - 63 \text{ X } 12}{16 \text{ X } 14 - 12 \text{ X } (38 + 47)} =$$

Gave Powers worksheet--worked some examples from it

--Tower of Hanoi problem (discussed and worked it)

Gave Patterns page 1 worksheet--worked some examples

Provided some time for students to do previous worksheets and get caught up

Did some interviewing

Week 6 (Lessons 16, 17, 18)

Worked on problems involving large numbers--most of these were adapted from those appearing in an article in the November 1977 Mathematics Teacher

--how to multiply or divide by 100, 1000, 1 million, etcetera, when the display is in scientific notation

Powers page--continued--worked some of the problems

Patterns pages 2 and 3--worked some examples

Reviewed constant of multiplication as done on the calculator

Week 7 (Lessons 19, 20, 21)

How to enter negative numbers into the calculator

Gave integers test (Grade 8 Mathematics--Appendix E)

--Had them do 3) 11) 17) 27) 30) 39) of the Grade 8 Mathematics test again showing the KSS and the answer

Gave the students Mid-project Student Survey to complete (Appendix E)

Gave them Patterns pages 4 and 5

Discussed the role of the calculator in their mathematics as a result of the Survey which they had just completed

Began with the work on Fractions--using the calculator to help with traditional algorithms

Week 8 (Lessons 22, 23, 24)

Discussed x^2 and y^x again

Gave "Talking" Calculator page (gave instructions and examples showing how this is to be done)

Gave 3 special problems on the chalkboard--students had to do them showing the KSS and the final answer

--one question involved finding the area of a city lot 18.6 m by 33.5 m

--the second asked for the cost of fencing the lot at $13.95 per metre

--the third was a calculation problem:

$$37^4 - (16 \times 14^5 - 8) + (8 \times 10^6) + 4000 =$$

--students took a long time to get these--had them do incorrect ones again after we had a discussion of errors

Did a number of interviews

Lost one class period at School A because of administrative rescheduling for examinations

At School B did more examples on previously given pages and discussed with them the relationship of calculations such as $A \div B \times C$ versus $A \times C \div B$ and showed that they were equal

Week 9 (Lessons 25, 26, 27)

Worked on Problems I sheet--gave them assistance

Gave Group Test No. 2 (were to give KSS and answer)

Interviewed a considerable portion of the time but interrupted this to assist individuals or the class with various exercises

Did percentage problems

Did conversions of numbers to percent, fractions to decimals

Did $\dfrac{63 \times 27}{933 + 44\ (17 + 3.8) - 234} =$ and various ways of doing this

Gave Fractions page 1 worksheet

Discussed Canadian Football field and calculated possible dimensions--implications for possible rule changes

Week 10 (Lessons 28, 29, 30)

Gave rest of Patterns pages and showed examples again of how students might approach such problems

--did adding of natural numbers up to any number

--beginning with 1

--beginning with any number other than 1

--sums of cubes of natural numbers beginning with 1

--did squares of 5, 15, 25, 35, 45, etcetera

--short way of multiplying by 11 (mentally--checked on the calculator--also used calculator to get the product and see if the pattern could be assessed)

--60 X 60 versus 61 X 59

--addition of odd natural numbers beginning with 1 and the relation to squares of natural numbers

Did how to get exact answers (in such cases as 2^{32} or in other calculations which go beyond the 8-digit normal capacity of the calculator)

Worked for a large part of the time on fractions

--doing them in fraction form (that is, having the answers in fraction form)

--how to reduce fraction by cancelling--using factoring in cancelling--using the calculator to simplify the factoring of any number

Did more of the questions from the Problems I page

Week 11 (Lessons 31, 32, 33)

Did Checkpoint Number 5 -- took the problems up with the whole class

Completed Problems I page

Continued with more work on fractions and the use of factoring in reducing final result to lowest terms

Worked examples from Fractions worksheets

--did fractions work in decimal form (not only converting mixed numbers to decimals but going on to do operations on such numbers)

Old fashioned problem (page 4 of Fractions worksheet)

--in fraction form--very lengthy and difficult

--also in decimal form--very much shorter and much easier to calculate

π-calculations from pages 4, 5, and 6 of the Fractions worksheets

Estimation Exercises (three estimations to be done for each question--largest selected--verification by use of the calculator)

Week 12 (Lessons 34, 35, 36)

Did Posttests--Canadian Test of Basic Skills (Mathematics Concepts and Mathematics Problem Solving)

How to find the repetand for fractions in which it is beyond the 8-digit capacity of the calculator (1/19, 1/29, 1/81)

Further π-calculations

Palindromes

Volume of sphere problem (page 7 of Fractions)

Puzzle problems (think of a number, double it, add 15, multiply result by 3, subtract 45, divide by 6 and what do you have? Answer: The number you started with)

--can you make some others of your own

--how does this look in algebraic form

--these are easy to follow on the calculator and can do several trials in a very short period of time

APPENDIX E

PRE- AND POSTTESTS,
CHECKPOINTS, GROUP TESTS, OPINION CHECKS

ADDITIONAL TEST GIVEN TO STUDENTS

Grade 8 Mathematics

1) $71 + 36 + {}^{-}42 =$ ____

2) $121 + {}^{-}215 + 36 =$ ____

3) $42 + {}^{-}36 + {}^{-}8 =$ ____

4) $65 + {}^{-}88 + {}^{-}3 =$ ____

5) ${}^{-}36 + {}^{-}17 + {}^{-}2 =$ ____

6) $36 + {}^{-}24 + 16 =$ ____

7) ${}^{-}36 + {}^{-}24 + 6 =$ ____

8) $42 + {}^{-}29 - {}^{-}3 =$ ____

9) $617 - {}^{-}312 =$ ____

10) ${}^{-}1006 - {}^{-}37 =$ ____

11) ${}^{-}316 - {}^{-}119 =$ ____

12) $412 - {}^{-}316 =$ ____

13) ${}^{-}338 - {}^{-}469 =$ ____

14) $216 + {}^{-}387 + {}^{-}138 + 216 + {}^{-}416 =$ ____

15) $({}^{-}84 - 21) + {}^{-}388 =$ ____

16) $61 - (41 - 21) =$ ____

17) $(-61 - {}^{-}38 - {}^{-}21 =$ ____

18) $-12 \text{ X } (-11 - {}^{-}61) =$ ____

19) $16 \text{ X } {}^{-}3 =$ ____

20) $-42 \text{ X } {}^{-}7 =$ ____

21) ${}^{-}111 \text{ X } 13 =$ ____

22) ${}^{-}14 \text{ X } {}^{-}13 \text{ X } {}^{-}12 =$ ____

23) -16 X 7 X $^{-}2$ = ____

24) $^{-}1$ X $^{-}43$ X $^{-}1$ X 19 = ____

25) $^{-}9102$ ÷ 74 = ____

26) 3228 ÷ -12 = ____

27) 03698 ÷ -43 = ____

28) -1736 ÷ -31 = ____

29) 1024 ÷ -64 = ____

30) -2583 ÷ -21 = ____

31) -6 + n = -3 ____

32) 43 - n = -16 ____

33) 6 X n = -42 ____

34) 3n + $^{-}2$ = -17 ____

35) -2n - 4 = 10 ____

36) n ÷ $^{-}3$ = 16 ____

37) n^2 - $^{-}3$ = ____ when n = -5

38) 17 + -6n = ____ when n = -2

39) a^2 - b = ____ when a = -2, b = -3

40) $5n^2$ - 7 = ____ when n = -1

Group Test No. 1

Do each question by showing KSS (Keystroke Sequence) and the final answer. The first is done for you. Underline KSS.

1. 75 + 69 = 75 + 69 = 144

2. 16 + (89 X 3) =

3. 387 - (466 + 243) =

4. 783.47 - 69.874 =

5. 7483 + [ ] = 4096

6. 643 + 929 + [ ] = 2617

7. 948 + 948 + 948 + 948 + 948 + 948 + 948 + 948 =

8. 8473 X 0 =

9. 1 X 7439 =

10. The product of 768 and 237

11. 0.010816 ÷ 0.00104 =

12. 888 X 888 ÷ 4.4 =

13. 7946 X (27.93 ÷ 0.3) =

14. $\dfrac{758 \times 4 \div 32}{16 \times 7 - 18 \div 64} =$

15. 460, 425.5, 391, 356.5, ____, ____, ____.

16. 3.7, 78.81, 1678.653, ____, ____, ____.

In the next one hit the target by multiplication.

17. 52, ____, ____, ____, ____, ____, ____, ____, 1726.4
(Place estimates in blanks)

Group Test No. 2 9th Week

For all of the following give me the KSS as well as the answer:

1) 7300 X (47 + 78) =

2) Do the first one again but this time get the correct answer without using parentheses or memory. (Give both KSS and answer)

3) Find the fifth power of 6.

4) Do 3) in a second way.

5) Do 3) again in a third way.

6) Express $\frac{23}{81}$ as a decimal.

7) Round the answer from 6) to nearest thousandth.

8) Express $\frac{23}{81}$ as a percent.

9) $\dfrac{63 \times 27}{933 + 44\,(17 + 3.8) - 234}$

10) Give answer from 9) correct to nearest tenth.

11) Do 9) another way showing a different KSS that will get the correct answer.

MID-PROJECT STUDENT SURVEY

1) Have you used your calculator outside of school (do not count your work on calculator work sheets which I gave you)? Yes ____

No ____

If you did, approximately how many times did you use it?

1___ 2___ 3 to 5___ 6 to 10___ more than 10 ____

If you did, explain how you used it and give examples of the sorts of questions you have done or for what sorts of topics you used the calculator.

2) Have you used your calculator in school in other subjects besides mathematics? Yes ____ No ____

If you did, approximately how many times did you use it?

1___ 2___ 3 to 5___ 6 to 10___ more than 10____

If you did, give examples of the kinds of questions you did even if you can't remember specific numbers.

3) Do you regularly take your calculator home? Yes ____ No ____ (Regularly means almost every night unless you just happen to forget). If you do tell me why you take it home:

If you do not tell me why you do not:

4) Do you usually carry your calculator with you throughout your school day? Yes ____ No ____

If you do tell me why you do:

If you do not tell me why you do not:

5) For what kinds of mathematics questions have you found that your calculator is the most useful?

6) From your work with the calculator so far would you recommend that all grade 8 students should have calculators for their personal use in school (don't worry about who would buy them)? Yes ____ No ____

Why or why not?

7) Can you name any problems that could happen if all grade 8 students had calculators at all times?

8) What kinds of things do you particularly like about the calculator you are using?

9) What kinds of things do you think are not very good about the calculator you are using?

10) Are there any kinds of calculations that you find difficult to do on your calculator?

11) If there are any other comments you wish to make about calculators and their use please do so:

CHECKPOINT NUMBER 1

1) Factor 60214

2) Multiply 589 764 by itself

3) Do $\frac{75 \text{ X } 93 \div 64 - 7}{38 + 16 \text{ X } 3}$

CHECKPOINT NUMBER 2

1) How would you do the following?

   a) 9 X 8 =

   b) 4 + 4 + 4 + 4 + 4 =

   c) 768 + 768 + 768 + 768 =

   d) 876 ÷ 100 =

   e) 2 476 345 X 1000 =

   f) 18 + 7 =

   g) 247 + 100 =

   h) 97 X 97 =

2) (86 X 86 + 47) X 25 + 17 =

3) Do this mentally:
   (4 + 6 X 3) X 3 + 4 =

   Write your answer.

   Now do it on the calculator.

   Did you get the same answer?

CHECKPOINT NUMBER 3

1) $14^4 - 2^{12} =$

2) Factor 9555.

CHECKPOINT NUMBER 4

1) Change $\frac{7}{8}$ to a decimal.

2) Change $\frac{9}{14}$ to a decimal and write it correct to the nearest thousandth.

3) Complete this fraction $\frac{14}{23} = \frac{\quad}{736}$.

4) Put the correct sign (>, <, or =) between these two fractions:

$$\frac{7}{12} \quad \frac{231}{396}$$

CHECKPOINT NUMBER 5

1) $\frac{63 \times 12 - 140}{114 - 73 \times 2} =$

2) $75 + 38 \times 4 - 16 \times 7 =$

3) $(75 + 38) \times 4 - 16 \times 7 =$

4) Are your answers for 2) and 3) different?

5) If the answers for 2) and 3) are different explain to me why this happens?

CHECKPOINT NUMBER 6 -- FINAL

1. By using the constant key do the following:

   a) 747, 723.4, 699.8, 676.2, _____, _____, _____

   b) 895 795.2, 149 299.2, 24 883.2, 4147.2, _____, _____, _____

2. a) Find 63% of 2467.

   b) What percent is 98 of 140?

   c) 78% of some number is 1833. What is that number?

3. a) If radio waves travel at a speed of 300 000 km/s how many seconds would it take a radio signal from earth to reach a place in space which is 13.5 billion km away?

   b) How many minutes is this?

   c) How many hours is this?

4. Change $18\frac{16}{23}$ to a fraction. $18\frac{16}{23} = \frac{\quad}{23}$

   Then change it to a new fraction as shown: $\frac{\quad}{23} = \frac{\quad}{322}$

5. Do this question in the usual fraction form:

   $16\frac{3}{7} + 38\frac{14}{19} =$

6. Do this question in the usual fraction form:

   $27\frac{4}{7} \times 36\frac{14}{15}$

7. Do this question in decimal form:

   $18\frac{7}{9} - 12\frac{10}{11} =$

8. Do this question in decimal form:

$$33\frac{7}{12} - 4\frac{15}{22} =$$

9. a) Estimate the answer to this question:

   67.73 X 38.5 =

   b) Calculate the answer for a).

   c) How much were you out from the actual answer?

   d) Is that more or less than 10% of the actual answer?

10. Just for something to do a grade 8 student decided to calculate how much it would cost him to have a hamburger everyday of the year for a whole year (365 days)
The hamburger he had in mind costs 85¢
He did the calculation and said it would cost him $31 025.
What would you say to him if he told you that?

11. a) Find, by trying out different values, what the answer is to the following:

    49 + 85 X [ ] = 703.5

    b) Calculate the answer for a) without guessing.

12. Do the following addition:

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} =$$

13. Find the area of a circle of radius of 16 cm.

    $(A = \pi r^2)$

14. Calculate the following:

$$^{-}64 + 198 \times {}^{-}32 - {}^{-}8344 \div 56 =$$

RATIONAL NUMBERS (Pretest)

PART I   Your Name ____________________

Circle the letter corresponding to your choice for the answer. Do your calculations on the paper provided.

1. $31.42 + 8.56 + 0.0134 + 21 =$

   A. 41.53   B. 60.9934   C. 39.9955   D. 50.9934

2. Another way to write 5 degrees below zero is

   A. 5° B. -5° C. $\frac{1}{5}^\circ$ D. $5^{-\circ}$

3. $-9 + {}^{+}43 =$

   A. -52 B. -34 C. $+34$ D. $+52$

4. An example of the additive identity property is

   A. $24 \times 0 = 0$ B. $36 + 0 = 36$ C. $0 \quad 4 = 0$

   D. $24 - 23 = 1$

5. The set of numbers which are both whole numbers and integers is

   A. $-3, 0, {}^{-}7, -8$ B. $-3, 0, {}^{-}7, {}^{+}8$

   C. $-3, 0, {}^{+}7, {}^{+}8$ D. $+3, 0, {}^{+}7, {}^{+}8$

6. If the additive inverse of a number is 5, then the sum of the additive inverse and the number is

   A. 0   B. $\frac{1}{5}$ C. 1   D. 10

7. $-25 - 8 =$

   A. -17 B. $+17$ C. -33 D. $+33$

8. Which of the following is NOT a rational number?

   A. 17   B. $4 \div 0$ C. $12 \quad 8$ D. -2

9. $\frac{-3}{4} + {}^{-}3 =$

   A. $\frac{-15}{4}$ B. $\frac{{}^{+}9}{4}$ C. $\frac{{}^{-}9}{4}$ D. $\frac{+15}{4}$

10. $\frac{-8}{5} - \frac{7}{3} =$

A. $\frac{-59}{15}$ B. $\frac{-56}{15}$ C. $\frac{-11}{15}$ D. $\frac{11}{15}$

11. Which of the following statements is true?

A. $\frac{432}{25} > 17.28$ B. $\frac{432}{25} = 17.28$ C. $\frac{432}{25} < 17.28$

d. $\frac{432}{25} = 17.7$

12. Which of the following is true?

A. $\frac{-1}{3} \times \frac{2}{3} = \frac{2}{3} \times \frac{1}{3}$ B. $\frac{-5}{8} \times \frac{3}{4} < \frac{-5}{8} \times \frac{1}{4}$

C. $\frac{-1}{8} \times \frac{4}{3} > \frac{-1}{8} \times \frac{1}{3}$ D. $\frac{1}{4} + \frac{-3}{5} = \frac{-3}{5} + \frac{-1}{4}$

13. The number $\frac{-7}{6}$ is the solution for the condition:

A. $\frac{1}{2}x + \frac{1}{4}x = \frac{-7}{8}$ B. $\frac{2}{3}x + \frac{-1}{6} = \frac{5}{12}$

C. $\frac{1}{6} + x = \frac{-3}{7}$ D. $\frac{1}{2} = x - \frac{-14}{6}$

14. What is the decimal equivalent of $\frac{5}{9}$?

A. $0.\overline{5}$ B. $1.\frac{4}{5}$ C. 1.8 D. $5.\overline{5}$

PART II

Do the calculations shown for 15. to 26.
Use the paper provided and hand that in with your test paper.

15. $161.4 \div 0.04 =$

16. $(34 + 88) + 30 \div 15 =$

17. $(-81 \div 3) \div (45 \div -5) =$

18. $-4 \times {}^{-}73 \times {}^{-}25 =$

19. ${}^{-}4\frac{1}{11} \times {}^{-}4\frac{8}{9} - 5\frac{3}{7} \times 1\frac{2}{19} =$

20. $$\begin{array}{r} 12\frac{1}{4} \\ 7\frac{1}{3} \\ +\ 8\frac{5}{6} \\ \hline \end{array}$$

21. $\frac{7}{8} - \frac{1}{3} =$

22. $\frac{4}{5} \times \frac{15}{16} =$

23. $1\frac{4}{5} \div 27 =$

24. $16\frac{3}{4} - 4\frac{5}{7} =$

25. $6\frac{7}{12} \times {}^{-}3\frac{9}{11} =$

26. $-18\frac{2}{5} \div {}^{-}12\frac{5}{8} =$

---

27. What is the fractional equivalent of -0.725?

28. Arrenge these numbers in ascending order:

    0.5 $\frac{1}{3}$ $\frac{5}{8}$ 0.0375

29. Find the value of $3p^2 - (9x - 6)$, if $p = 3$ and $x = 4.3$.

30. If $14X - 2\frac{1}{2} = 8x + -2\frac{1}{2}$, then what is x equal to?

31. If $\frac{1}{2}x = \frac{24}{5}$, then what is x equal to?

32. Solve for x; $\frac{1}{2}x + \frac{-3}{x}x = 2\frac{1}{2}$

33. Solve for n; $-7 \times \frac{-3}{10} \times -10 = 7 \times n \times \frac{-3}{10}$

34. Are the following fractions equivalent? Show your work.

    $\frac{15}{17}$ and $\frac{16}{19}$

35. What number when divided by 8 yields a quotient of 15?

36. The sum of three numbers is 132. The second number is 4 times the first and the third is 7 times the first. Find the numbers.

RATE, RATIO AND PERCENT (Pretest)

PART I Your Name ____________________

Circle the letter corresponding to your choice for the answer. Do your calculations on the paper provided.

1. If $\frac{16}{40} = \frac{x}{100}$, then x is

   A. 96 B. 52 C. 40 D. 250

2. If 9 : 36 = x : 100, then x is

   A. 25 B. 40 C. 46 D. 73

3. 1½ expressed as a percent is

   A. $\frac{3}{2}$ B. 1.5% C. 15% D. 150%

4. As a decimal $33\frac{1}{3}$ percent is

   A. $\frac{1}{3}$ B. 0.3 C. $0.0\overline{3}$ D. $33.\overline{3}$

5. ½% of 2000 is

   A. 10 B. 100 C. 1000 D. 10 000

6. John gets an allowance of $8.80. If he spends $\frac{1}{5}$ of it then the amount he still has is

   A. $1.76 B. $7.04 C. $2.20 D. $6.60

7. A flag pole 15 m long cast a shadow 25 m long. If the shadow cast by a spruce tree, at the same time, is 40 m long, the tree height is

   A. 9.375 m B. 24 m C. $66.\overline{6}$ m D. 4.8 m

8. Mary received 60% on a test. If there were 50 questions on the test the number of questions Mary had correct was

   A. 30 B. $83\frac{1}{3}$ C. 120 D. 40

PART II

Do the following questions on the paper provided and hand that in with your test paper.

9. Convert $\frac{54}{625}$ to a percent.

10. Convert 240% to the equivalent basic fractional form.

11. Express 12 as a percent.

12. Express 0.023 as a percent.

13. Express 0.05 percent as a decimal.

14. Solve for n ; n% of 36 is 9

15. Solve for n ; 40% of n is 62

16. Solve for n ; 112% of 6 is n

17. Solve for n ; 65 is n% of 60

18. Suppose that the Progressive Conservative party won 4 out of 5 seats in the 265 seat House of Commons. How many seats did they win?

19. Canada has a population of approximately 22 000 000. If 30% of Canada's population is bilingual, how many Canadians speak two languages?

20. My two year old sister takes 9 steps to go as far as I do in 2 steps. At this rate how many steps will she have to take to cover the distance I go in 360 steps?

21. In a school election, Dan received five votes to every two votes Andy received. If Andy received 120 votes, how many did Dan receive?

22. After travelling 600 km we had covered 60% of our trip. How many kilometres will we travel in the whole trip?

RATIONAL NUMBERS (Posttest)

PART I Your Name ____________________

Circle the letter corresponding to your choice for the answer. Do your calculations on the paper provided.

1. 41.62 + 9.73 + 0.0245 + 23 =

   A. 76.80 B. 74.3745 C. 51.3768 D. 41.6045

2. Jim has $26 in a bank account and writes a cheque for $29. His bank balance can be represented by

   A. 29 - 26 B. 26 - 29 C. 26 - $^{-}$29 D. -26 - 29

3. -24 + -48 + 34 =

   A. -106 B. -38 C. +38 D. +106

4. The identity element for multiplication is illustrated in:

   A. 15 X 0 = 0 B. 15 X a = 0

   C. 15 X a = a D. 15 X 1 = 15

5. The set of whole numbers does NOT include which of the following sets:

   A. 0 B. 0, 1, 2, 3, ... C. ..., -3, -2, -1

   D. 1

6. The expression which illustrates the additive inverse property is

   A. $-4 + \frac{-1}{4} = 0$ B. $4 + \frac{-1}{4} = 0$ C. -4 + $^{-}$4 = 0

   D. 4 + -4 = 0

7. -24 + -48 - 34 =

   A. -38 B. -106 C. 38 D. 106

8. Which of the following is NOT a rational number?

   A. 23 B. 6 ÷ 0 C. 15 ÷ 5 D. -4

9. $-1\frac{1}{3} + 3\frac{1}{2} =$

A. $-4\frac{1}{6}$ B. $-2\frac{1}{6}$ C. $+4\frac{1}{6}$ D. $+2\frac{1}{6}$

10. $-\frac{22}{7} - \frac{-3}{14} =$

A. $\frac{-41}{14}$ B. $\frac{-47}{14}$ C. $+\frac{25}{14}$ D. $\frac{19}{14}$

11. Which of the following statements is true?

A. $\frac{461}{25} > 18.44$ B. $\frac{461}{25} = 18.44$ C. $\frac{461}{25} < 18.44$

D. $\frac{461}{25} = 18.4$

12. Which of the following is true?

A. $\frac{-1}{7} \times \frac{2}{7} = \frac{2}{7} \times \frac{1}{7}$ B. $\frac{-7}{8} \times \frac{3}{4} < \frac{-7}{8} \times \frac{1}{4}$

C. $\frac{-1}{10} \times \frac{5}{3} > \frac{-1}{10} \times \frac{4}{3}$ D. $\frac{1}{4} + \frac{-3}{5} = \frac{-3}{5} + \frac{-1}{4}$

13. The number $\frac{-7}{6}$ is the solution for the condition:

A. $\frac{1}{2}x + \frac{1}{4}x = \frac{-7}{8}$ B. $\frac{2}{3}x + \frac{-1}{6} = \frac{5}{12}$

C. $\frac{1}{6} + x = \frac{-3}{7}$ D. $\frac{1}{2} = x - \frac{-14}{6}$

14. What is the decimal equivalent of $\frac{4}{9}$?

A. $0.\overline{4}$ B. $1.\frac{3}{4}$ C. 1.75 D. $4.\overline{4}$

PART II

Do the calculations shown for 15. to 26.
Use the paper provided and hand that in with your test paper.

15. $151.2 \div 0.03 =$

16. $16 + 38 \div 2 - 12 =$

17. $(-84 \div 3) \div (42 \div -6) =$

18. $+6 \text{ X } {}^-78 \text{ X } {}^-44 =$

19. ${}^-4\frac{2}{7} \text{ X } {}^-4\frac{2}{3} \; - \; 5\frac{3}{7} \text{ X } 1\frac{2}{19} =$

20. $$\begin{array}{r} 13\frac{1}{4} \\ 8\frac{5}{6} \\ +\ 9\frac{1}{3} \\ \hline \end{array}$$

21. $\frac{7}{8} - \frac{1}{3} =$

22. $\frac{3}{4} \text{ X } \frac{16}{21} =$

23. $1\frac{5}{7} \div 36 =$

24. $18\frac{3}{4} - 6\frac{5}{7} =$

25. $7\frac{5}{12} \text{ X } {}^-4\frac{8}{11} =$

26. $-19\frac{3}{5} \div {}^-16\frac{5}{8} =$

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27. What is the fractional equivalent of -0.675?

28. Arrange these numbers in ascending order:

$$0.7 \qquad \frac{5}{9} \qquad \frac{7}{8} \qquad 0.0475$$

29. Evaluate $4m - 14m$, if $m = \frac{4}{5}$

30. If $27x - 6\frac{3}{4} = 12x + -6\frac{3}{4}$, then what is x equal to?

31. If $\frac{1}{5}x = \frac{25}{3}$, then what is x equal to?

32. Solve for x; $\frac{1}{4}x + \frac{-3}{8}x = 2\frac{5}{8}$

33. Solve for n; $-11 \times \frac{-3}{14} \times -14 = 11 \times n \times \frac{-3}{14}$

34. Are the following fractions equivalent? Show your work.

$$\frac{17}{21} \text{ and } \frac{18}{23}$$

35. What number when divided by 7 yields a quotient of 13?

36. The sum of three numbers is 154. The second number is 3 times the first and the third is 7 times the first. Find the numbers.

RATE, RATIO AND PERCENT (Posttest)

PART I

Your Name ____________________

Circle the letter corresponding to your choice for the answer. Do your calculations on the paper provided.

1. If $\frac{3}{7} = \frac{x}{147}$, then x is

   A. 147 B. 143 C. 63 D. 21

2. If 8 : 32 = x : 160, then x is

   A. 40 B. 20 C. 45 D. 5

3. $1\frac{3}{4}$ expressed as a percent is

   A. $\frac{7}{4}$ % B. 1.75% C. 17.5% D. 175%

4. As a decimal $66\frac{2}{3}$ percent is

   A. $\frac{2}{3}$ B. $0.\overline{6}$ C. $66.\overline{6}$ D. $0.0\overline{6}$

5. $\frac{1}{3}$ % of 30 000 is

   A. 10 B. 100 C. 1000 D. 10 000

6. Jim earned $9.40. If he spends ¼ of it, then he still has

   A. $2.35 B. $7.05 C. $1.88 D. $7.52

7. There are 28 students in the mathematics class. The boys outnumber the girls 3 to 1. The number of boys in the class is

   A. 20 B. 18 C. 15 D. 21

8. Joan received 70% on a test. If there were 80 questions on the test, the number of questions Joan had correct was

   A. 56 B. $114\frac{2}{7}$ C. 160 D. 24

PART II

Do the following questions on the paper provided and hand that in with your test paper.

9. Convert $\frac{35}{112}$ to a percent.

10. Convert 540% to the equivalent basic fractional form.

11. Express 16 as a percent.

12. Express 0.034 as a percent.

13. Express 0.04 percent as a decimal.

14. Solve for n ; n% of 48 is 12

15. Solve for n ; 50% of n is 74

16. Solve for n ; 118% of 8 is n

17. Solve for n ; 95 is n% of 90

18. Suppose that Edmonton Oilers won $\frac{5}{7}$ of their league games. In a seventy game schedule how many games did they win?

19. In 1961, 45% of Alberta's population of 1 332 000 were of British origin. How many people were of British origin?

20. An airplane travels 58 km in the time our car goes 7 km. How far could we go in our car in the time that the airplane takes to go 4640 km?

21. Mary earns seven dollars for every four dollars that her sister Myrna earns. If Myrna earns $148, how much will Mary earn in that time?

22. After travelling for 900 km we had covered 80% of our trip. How many kilometres will we travel in the whole trip?

ADDITION CHECK

Work down the columns.
Do not omit any.

Time: 1 min

2 + 7 =
8 + 4 =
9 + 6 =
9 + 9 =
3 + 4 =

6 + 7 =
8 + 8 =
2 + 1 =
0 + 7 =
9 + 5 =

1 + 3 =
4 + 4 =
8 + 2 =
3 + 7 =
4 + 5 =

9 + 2 =
8 + 9 =
7 + 4 =
6 + 1 =
7 + 3 =

9 + 9 +
8 + 1 =
3 + 3 =
5 + 6 =
6 + 8 =

2 + 7 =
6 + 4 =
7 + 8 =
9 + 8 =
7 + 2 =

4 + 5 =
6 + 7 =
1 + 7 =
8 + 9 =
2 + 4 =

8 + 3 =
3 + 2 =
7 + 6 =
4 + 9 =
6 + 0 =

8 + 2 =
5 + 1 =
5 + 7 =
4 + 8 =
6 + 6 =

7 + 9 =
3 + 1 =
8 + 2 =
2 + 4 =
9 + 6 =

1 + 8 =
9 + 3 =
2 + 1 =
8 + 4 =
3 + 5 =

7 + 9 =
4 + 6 =
6 + 8 =
5 + 5 =
7 + 8 =

6 + 8 =
4 + 5 =
7 + 7 =
3 + 9 =
8 + 8 =

2 + 6 =
9 + 2 =
6 + 4 =
7 + 8 =
1 + 3 =

2 + 3 =
4 + 7 =
6 + 2 =
9 + 8 =
1 + 7 =

SUBTRACTION CHECK

Work down the columns.
Do not omit any.

Time: 1 min

12 - 4 =
16 - 7 =
9 - 3 =
8 - 4 =
17 - 8 =

15 - 6 =
18 - 9 =
7 - 4 =
17 - 8 =
6 - 2 =

11 - 3 =
5 - 2 =
13 - 7 =
8 - 0 =
13 - 9 =

6 - 5 =
12 - 7 =
10 - 4 =
15 - 7 =
5 - 3 =

11 - 7 =
8 - 4 =
3 - 2 =
16 - 8 =
13 - 6 =

4 - 3 =
13 - 4 =
11 - 2 =
8 - 6 =
16 - 8 =

12 - 9 =
14 - 7 =
15 - 8 =
14 - 8 =
10 - 5 =

13 - 7 =
9 - 5 =
8 - 5 =
10 - 4 =
9 - 2 =

14 - 6 =
16 - 9 =
8 - 3 =
12 - 9 =
3 - 2 =

8 - 1 =
9 - 8 =
15 - 6 =
16 - 9 =
11 - 4 =

10 - 5 =
11 - 2 =
9 - 4 =
12 - 6 =
2 - 1 =

9 - 0 =
8 - 8 =
15 - 7 =
8 - 3 =
4 - 2 =

12 - 6 =
16 - 9 =
13 - 9 =
8 - 7 =
14 - 6 =

15 - 7 =
14 - 8 =
12 - 8 =
6 - 2 =
7 - 0 =

15 - 8 =
9 - 4 =
12 - 7 =
13 - 4 =
7 - 4 =

MULTIPLICATION CHECK

Work down the columns.
Do not omit any.

Time: 1 min

1 X 2 =
3 X 4 =
4 X 1 =
5 X 6 =
2 X 4 =

8 X 3 =
6 X 7 =
9 X 8 =
8 X 7 =
9 X 3 =

6 X 9 =
5 X 8 =
4 X 7 =
3 X 2 =
1 X 7 =

2 X 9 =
3 X 9 =
6 X 5 =
3 X 3 =
4 X 9 =

7 X 7 =
2 X 2 =
3 X 5 =
5 X 9 =
6 X 4 =

8 X 7 =
9 X 1 =
2 X 8 =
6 X 8 =
7 X 9 =

6 X 6 =
1 X 4 =
0 X 8 =
3 X 6 =
8 X 2 =

7 X 7 =
2 X 3 =
4 X 4 =
7 X 0 =
6 X 3 =

4 X 1 =
6 X 9 =
8 X 7 =
9 X 8 =
7 X 2 =

4 X 3 =
1 X 8 =
3 X 3 =
7 X 6 =
3 X 9 =

6 X 8 =
9 X 0 =
2 X 1 =
6 X 4 =
8 X 4 =

7 X 3 =
4 X 8 =
3 X 4 =
5 X 6 =
4 X 3 =

2 X 7 =
4 X 5 =
6 X 6 =
8 X 3 =
7 X 1 =

9 X 8 =
6 X 3 =
2 X 9 =
7 X 7 =
6 X 1 =

4 X 7 =
3 X 2 =
6 X 9 =
8 X 7 =
2 X 3 =

DIVISION CHECK

Work down the columns.
Do not omit any.

Time: 1 min

3 ÷ 1 =
6 ÷ 3 =
4 ÷ 2 =
12 ÷ 3 =
16 ÷ 4 =

42 ÷ 7 =
81 ÷ 9 =
25 ÷ 5 =
48 ÷ 6 =
8 ÷ 4 =

7 ÷ 7 =
15 ÷ 3 =
9 ÷ 3 =
14 ÷ 2 =
54 ÷ 6 =

8 ÷ 1 =
30 ÷ 5 =
24 ÷ 6 =
12 ÷ 4 =
2 ÷ 1 =

21 ÷ 3 =
6 ÷ 6 =
18 ÷ 2 =
64 ÷ 8 =
35 ÷ 7 =

16 ÷ 8 =
36 ÷ 9 =
8 ÷ 1 =
24 ÷ 4 =
40 ÷ 5 =

81 ÷ 9 =
72 ÷ 8 =
35 ÷ 7 =
36 ÷ 6 =
54 ÷ 9 =

42 ÷ 6 =
9 ÷ 9 =
25 ÷ 5 =
20 ÷ 4 =
24 ÷ 8 =

27 ÷ 3 =
10 ÷ 2 =
6 ÷ 3 =
56 ÷ 7 =
54 ÷ 6 =

30 ÷ 5 =
18 ÷ 9 =
7 ÷ 1 =
0 ÷ 3 =
28 ÷ 4 =

27 ÷ 3 =
63 ÷ 7 =
24 ÷ 6 =
28 ÷ 4 =
81 ÷ 9 =

64 ÷ 8 =
63 ÷ 9 =
16 ÷ 2 =
18 ÷ 6 =
10 ÷ 5 =

32 ÷ 8 =
56 ÷ 7 =
72 ÷ 9 =
49 ÷ 7 =
42 ÷ 6 =

63 ÷ 7 =
35 ÷ 5 =
9 ÷ 1 =
10 ÷ 2 =
14 ÷ 7 =

16 ÷ 8 =
12 ÷ 6 =
56 ÷ 7 =
63 ÷ 9 =
15 ÷ 5 =

ESTIMATION CHECK

1. For each of the following give your estimation of the right answer. A few examples are given.

Examples:		
16 + 28	45	(thinking 15 + 30)
8 X 43	320	(thinking 8 X 40)
9 X 67	630	(thinking 9 X 70)
	30	(thinking 70 - 40 or 78 - 48 or 80 - 50)

You do not need to write down why you estimated what you did:

	Estimate		Estimate
1. 2 X 29		2. 80 + 190	
3. 103 - 69		4. 39 ÷ 2	
5. 75 X 9		6. 19 X 7	
7. 604 - 208		8. 247 + 261	
9. 95 X 69		10. 18 X 9	
11. 37 + 98 + 57		12. 1283 - 879	
13. 29 X 11		14. 78 ÷ 4	
15. 804 ÷ 39		16. 766 - 281	
17. 237 X 42		18. 1035 ÷ 25	
19. 33 ÷ 6		20. 297 X 39	
21. 318 + 347		22. 98 ÷ 5	
23. 198 X 7		24. 378 - 247	
25. 43 X 78			

FINAL ESTIMATION CHECK -- No. 2

1. 34 X 65 =
2. 98 + 76 =
3. 750 - 260 =
4. 85 X 16
5. 55 X 48 =
6. 8473 + 4763 =
7. 64 ÷ 36 =
8. 15 X 9 X 7 =
9. 545 ÷ 51 =
10. 1000 - 489 =
11. 7054 ÷ 120 =
12. 378 X 5 =
13. 898 + 348 =
14. 78 + 89 X 9 =
15. 97 X 89 =
16. $204\overline{)1700}$ =
17. 4 X 4 X 4 X 4 X 4 X 4 =
18. 17 + 89 + 27 =
19. 483 - 47 - 56 =
20. 48 X 48 =
21. 35 ÷ 21 =
22. 197 X 0.5 X 18 =
23. 99 ÷ 3.8 =
24. 470 X 4.9 =
25. 1947 ÷ 2 =
26. 86 X 28 =
27. 95 + 90 =
28. 1176 - 445 =
29. 777 X 6 =
30. 21 ÷ 11 =
31. 475 ÷ 23 =
32. $12\overline{)820}$ =
33. 153 X 21 =
34. 1900 - 240 =
35. 3 X 3 X 57 =
36. 148 + 517 =
37. 88 X 24 =
38. 65 X 9 X 97 =
39. 78 X 19 ÷ 8 =
40. 1583 - 1460 + 235 =
41. 247 + 162 =
42. 42 X 32 =
43. 78 ÷ 2.1 =
44. 89 X 1.9 =
45. 70 - 30.753 =
46. 115 + 84.315 =

47. 1543 - 750 =

48. 88 X 8.865 =

49. 48 + 48 + 48 =

50. $29\overline{)3000}$ =

OPINION CHECK

Underline the one that you think best describes your feelings. Since this depends on your feelings, there are not right or wrong answers. Just give your honest opinion.

SD -- means Strongly Disagree
D -- means Disagree
U -- means Undecided
A -- means Agree
SA -- means Strongly Agree

1. I enjoy working on my mathematics homework.
SD D U A SA

2. Mathematics is an enjoyable subject to me.
SD D U A SA

3. Mathematics makes me feel uneasy and confused.
SD D U A SA

4. I like to do mathematical questions in other subjects and outside of school.
SD D U A SA

5. When we have to do mathematics, I get nervous and uncomfortable
SD D U A SA

6. I have always liked mathematics in school.
SD D U A SA

7. Mathematics is a dull and boring subject.
SD D U A SA

8. I am interested in studying more mathematics in Senior High School.
SD D U A SA

9. I have never liked mathematics and it is my most dreaded subject.
SD D U A SA

10. I am interested and willing to take more mathematics when I shall have the opportunity.
SD D U A SA

11. Mathematics is not a very practical subject and we should do less of it in school.
SD D U A SA

12. I like to work on mathematics and help other students with it.
SD D U A SA

13. I enjoy trying difficult questions in mathematics whenever these come up in class.
SD D U A SA

OPINION CHECK --- Parents

This survey is an attempt to obtain your present opinion on the use of minicalculators in the school. You are being asked these questions because your child is participating in a project using a minicalculator in mathematics. Your opinions will help form an overall assessment on the status of minicalculators in the schools.
Names of students or parents will not appear in any final statements that may be made as a result of this project.

1. Do you have any calculators in your home at present?
   Yes _____ No _____

2. If you do have any calculator(s), is your child allowed to use them (it) if he or she wishes?
   Yes _____ No _____

   If there are any restrictions on its use, please indicate these.

3. In general do you favor the use of minicalculators to some extent in Junior High School?
   Yes _____ No _____

4. In what way might the use of the minicalculator in the grade 8 classroom affect your child's performance in basic computational skills?
   Hinder _____ Have little or no effect ___
   Improve ____ No opinion _____

5. In what way might the use of the minicalculator in the grade 8 classroom affect your child's overall performance in mathematics?
   Hinder _____ Have little or no effect ___
   Improve ____ No opinion ____

6. In what way might the use of the minicalculator in the grade 8 classroom affect your child's attitude towards mathematics?
   Hinder _____ Have little or no effect ___
   Improve ____ No opinion _____

7. Will the use of the minicalculator in the grade 8 classroom make your child dependent on it in the performance of basic skills?
   Highly dependent ____ Dependent to a slight degree ____
   Not dependent ____ No opinion ____

8. Should the Edmonton Public School System permit the use of the minicalculator in the grade 8 classroom?
   Yes ____ No ____ No opinion ____

9. If you said "Yes" to number 8, should the minicalculator be permitted for all subjects?
   Yes ____ No ____ No Opinion ____

10. If you said "Yes" to number 8, should the minicalculator be permitted when students are writing tests?
    Yes ____ No ____ No opinion ____

11. Should the Edmonton Public School System provide instruction to its students in the use of the minicalculator?
    Yes ____ No ____ No opinion ____

12. If you said "Yes" to number 11, in what grade do you think instruction should begin?
    12 ___ 10 ____ 8 ____ Below 7 ____
    11 ___ 9 ____ 7 ____

13. If you wish, make any comments you like about the use of minicalculators in the school.

APPENDIX F

CALCULATOR STUDENTS

Student (by Code Number)	Age (Start of Project)	IQ*	Final Grade 7 School Mathematics Mark (%)
F - 111	13-11	126	75
F - 112	13-1	108	45
M - 113	13-11	106	70
F - 114	14-1	67	35
M - 115	13-3	135	70
F - 116	13-3	92	60
M - 117	13-8	89	60
M - 118	13-6	98	45
F - 119	13-5	110	80
F - 120	15-0	75	42
M - 121	15-4	83	C (in Special Ed. class)
M - 122	13-6	129	58
M - 123	13-7	108	69
M - 124	13-4	102	78
M - 125	14-9	85	59
F - 126	14-1	96	82

*Canadian Lorge Thorndike Test

APPENDIX G

OBSERVATIONS FROM THE PILOT STUDY

OBSERVATIONS FROM THE PILOT PROJECT

The following observations, presented in no particular order, were made as a result of the experiences during the pilot study:

1. Even though students have been exposed to calculators and may even have them at home, they do not know how to use them efficiently. They seem to know only how to carry out the four basic operations.

2. Several students commented that many questions were boring because they were so simple. It appears that there is simply no reason for giving students a number of repetitive exercises. Even a lengthy puzzle problem (such as one similar to a cross-word puzzle but involving mathematical calculations) seems to be boring because of the simplicity.

3. Good students tended to compete for speedy solutions and thereby often made errors.

4. Good students seemed most keen when there appeared to be some challenge; for example, decide how this series is formed and fill the blanks with correct numbers to complete the series:

59 049, 19 683, 6561, 2187 _____, _____, _____, _____, 3.

5. Estimation is not as well developed a skill in grade 8 as are other skills in mathematics.

6. Students are reluctant to analyze situations; rather they seem to want procedures given to them directly.

7. All students, but especially slow students, will need to be given careful instructions and opportunities for practice in order to become proficient in using a calculator. The usual procedures of good teaching apply.

8. It takes time (more than two weeks) for most students to appreciate the power of the calculator.

9. The efficient use of the calculator is not a triviality; there is a significant amount of learning, not just in the manipulation of the keys to effect a solution, but to do so with ease and mathematical eloquence. For example, $\frac{75}{16 \times 34}$ should be done by 75 ÷ 16 ÷ 34 = but no students using the calculator in the early stages would do that. Rather they would likely do 16 X 34 = , record the result, and then do 75 ÷ (recorded result) = . Later students tend to make use of the calculator memory rather than record an intermediate result. However, the efficient way requires some analysis and application of mathematical knowledge.

10. In general students are enthusiastic about the opportunity to use calculators; other students express interest in their work.

11. The teacher reported that she noticed the ease with which so-called problems became trivial calculations to the students who had calculators.

12. No problems occurred in view of the fact that students were allowed to carry their calculators wherever they wished and that they were encouraged to use them.

13. The cassette recorder provides a good record of the class talk.

14. It may be necessary to arrange time from regular mathematics periods for personal interviews. The time in the calculator group sessions appeared to pass very quickly leaving little time for personal interviews of students by the researcher.

15. One parent is very much in favor of the use of calculators, two were not certain with a number of reservations, and one was opposed. Two did not complete the Opinion Check.

16. Relatively inexpensive 9-volt batteries will not last much more than two weeks--hence they are a nuisance and a waste of money. As a result it was decided that good quality batteries (usually described as heavy duty) would be utilized throughout the project.

APPENDIX H

ADDING FRACTIONS WITH THE CALCULATOR

ADDING FRACTIONS WITH THE CALCULATOR

Consider a question like $\frac{115}{37} + \frac{28}{63}$. With the power of just a $5 calculator -- basic functions and memory -- the solution is trivial:
115 ÷ 37 = (M) 28 ÷ 63 = + (MR) = ; this yields a decimal result. (M) means that the displayed value will be placed in Memory and (MR) refers to a key which recalls from Memory. However if a teacher wants a fractional form, a student can use his calculator to find very quickly that the result is $\frac{8281}{2331}$ and furthermore by dividing 8281 by 2331 he realizes that the fraction is equal to 3 + a fraction which he can readily obtain: $\frac{8281}{2331} = 3\frac{1288}{2331}$. With some understanding of his calculator the student can soon establish a method of deciding if the fraction is in lowest terms. He can factor the numbers quickly to obtain

$1288 = 2 \times 2 \times 2 \times 7 \times 23$ and $2331 = 3 \times 3 \times 7 \times 37$.

From this the final result is $\frac{115}{37} + \frac{28}{63} = 3\frac{184}{333}$.

Although this process appears lengthy and complicated in the explanation, it is relatively simple in the execution. Teachers would hesitate to give students such a question for paper and pencil calculation because of the high chance of error but on a calculator the manipulation of these numbers is only slightly more complicated than for fractions involving simpler numbers.

APPENDIX I

FOLLOW-UP CHECKPOINT

FOLLOW-UP CHECKPOINT

1. $764 + 89 \times 17 =$

2. $37 \times 46 + 37 \div 16 =$

3. $(38 + 97) \times 12 =$

4. $0.0127 \div 0.00125 =$

5. $\dfrac{65 \times 12 + 4}{16 - 7 \times 18 \div 25} =$ Give answer correct to nearest thousandth

6. $23 \times 23 \times 23 \times 23 \times 23 =$

7. Multiply each of the following by 12:

   7, 38, 49, 2374

7. Divide each of the following by 32:

   12, 78, 97, 441

9. $\frac{1}{17} =$ $\frac{1}{45} =$ $\frac{1}{625} =$

10. Complete this fraction $\frac{15}{22} = \frac{\quad}{264}$

11. Do these in decimal form:

    a) $34\frac{7}{9} - 15\frac{10}{11} =$

    b) $75\frac{4}{7} \div 30\frac{2}{13} =$

12. a) Find the square root of 799.1929

    b) Check your answer.

13. Do $(17.5 \times 10^{10}) \div (8.5 \times 10^{6}) =$ Write the answer in scientific notation

14. Do 39.4 billion multiplied by 6.4 million and write the answer in scientific notation.

APPENDIX J

EXCERPTS ILLUSTRATING DIFFICULTIES IN MATHEMATICS

Excerpts on working a rational number question, $16\frac{3}{7} + 38\frac{14}{19} =$. Two students had trouble after they got to $54\frac{155}{133}$. Although the problem is not with the calculator, the following excerpts show the mathematical difficulties.

Excerpt No. 1 (M-117)

[Student had $54\frac{155}{133}$]

R: What happens now?

S: You reduce them.

R: Right [Student did 133 ÷ 11 =]

Isn't $\frac{155}{133}$ bigger than 1?

S: Yes.

R: So don't you have to change $\frac{155}{133}$ so you can get a whole number to be added to 54? [No reply]. How many times does 133 divide 155? [Student proceeded to key 133 ÷ 155 into the calculator]

That's the wrong order. You're entering 133 divided by 155.

S: I always get mixed up.

R: Just remember to start at the top, the numerator, and just enter them in that order.

S: Uh, huh.

R: Actually you don't even need the calculator. Just look at the two numbers. How many times does 133 divide 155? [Pause - no reply, then he did 155 - 133 =].

Well look at the answer; what is the whole number?

S: 1

R: And how many 133's left over? [No reply]

How many one hundred thirty-thirds does it take to make 1?

S: 1

R: You mean $\frac{1}{133}$ makes 1?

S: No

R: That's what I mean. How many does it take to get 1?

S: 155

R: No [researcher then led him through several steps to get $\frac{155}{133} = \frac{133}{133} + \frac{\quad}{133}$, supplying the 22 for the second fraction. Even though this led to $1\frac{22}{133}$, the researcher had to supply $55\frac{22}{133}$ as the student did not]

How can we tell if $\frac{22}{133}$ is in its simplest form? [No reply]

What can you divide 22 by?

S: 133

R: No. What numbers will divide into 22 and give no remainders?

S: 2

R: And what else? [No reply]. Well 22 is 2 times what?

S: 11

R: Can you break 11 down into smaller numbers?

S: No.

R: So all I need to do to see if I can reduce the fraction is to see if I can divide what? [No reply]. Well 22 is 2 X 11 so what do I check?

S: 2 into 133.

R: Does it divide? [Again student did 2 ÷ 133 =]

But that sign is not a divide into sign [so researcher did it for student on student's calculator]

What else do you check?

S: 11 [Student did 133 ÷ 11 = and saw it does not divide].

R: So what can you say about the fraction?

S: It stays the same.

Excerpt No. 2 (F-120):

[Student had $54\frac{155}{133}$]

R: What will this be? [$\frac{155}{133}$] [No reply]

Well $\frac{155}{133}$ gives you how many whole ones? [No reply]. Is that [$\frac{155}{133}$] more than 1? [No reply]. It is equal to what? [No reply]. That is one since you can take $\frac{133}{133}$ out of it to make one and then what is left? [No reply]. Well, you subtract them, 155 minus 133, and what is left?

S: 22.

R: Yes, so now we have 54 and that much more [showing $1\frac{22}{133}$]

How much is that?

S: You add them.

R: And get what?

S: $55\frac{22}{133}$

R: Good. What can you divide 22 by?

S: 2

R: 2 and what else?

S: 4

R: No. 2 times what is 22?

S: 11

R: Right, and 11 is a prime number so can you divide 133 by 2? [No reply]

You can try it on the calculator to see if you can divide. [Student did that].

S: No. [Then tried 133 ÷ 11 =]

R: Can you divide it by 11?

S: No.

R: So that's it; $\frac{22}{133}$ can't be reduced to lower numbers.

[Researcher's final note in the transcript was that he doubts that the student really understood what was attempted but he did not pursue this further].

Excerpt 3 (School B - Rational Numbers):

[After a few questions had been done this one was given. It indicates what happens with simple questions. The question was $\frac{16}{33} \div \frac{6}{11} =$]

R: What should we do here?

Several students: Invert $\frac{6}{11}$ and multiply.

[Researcher wrote $\frac{16}{33} \times \frac{11}{6}$ and reduced this by cancellation upon suggestion of various students. Result was $\frac{4}{9}$ and no calculator work was used]

Excerpt 4 (School B - Rational Numbers):

[Problem was $95\frac{17}{37} \div 12\frac{65}{73} =$]

R: What will we get here?

M-124: [After calculations] $\frac{3532}{37}$

F-119: times $\frac{73}{941}$ [student did the calculation and the necessary inversion]

R: Good and that leads to what?

F-119: 257836

R: Over?

F-126: 34817

M-124: We should divide to get the whole number.

[The fraction was $\frac{257836}{34817}$]

[The division was performed and the integral portion was reported as 7].

R: Now what is the next step?

M-121: [He was always able to see this procedure].

7 X 34817 = +/- +257836 =

[He did this and reported 14117]

R: What is our answer to this point?

F-126: $7\frac{14117}{34817}$

[At this point went through another example of converting $\frac{119}{12}$ to $9\frac{11}{12}$ to see the algorithms used. The whole procedure is 119 ÷ 12 = to get 9.916667, follow this with 9 X 12 = (9 is the integral part of the number) to get 108 and then subtract 108 from 119 mentally or with 108 in display continue with +/- +119 = .]

[After above example class returned to problem of seeing if $\frac{14117}{34817}$ can be reduced].

R: Factor 14117 by the method we used.

M-124: 19 X 743 [this after suitable interval of calculation]

R: Good. Let us see if 743 can be factored. How far will we need to go before we quit checking?

M-124: To the square root of 743.

R: Very good and where will we start?

M-124: at 23.

R: Why is that so?

M-124: Well, we already did up to 19 so we just keep going from there.

[Class established that 19 X 743 is in prime factors]

R: How do I check if we can reduce the fraction?

M-124: Try to divide 24817 by 7 and by 743.

R: Good. Do that.

[Class did this and found that the fraction could not be reduced to lower terms].

It should be noted that the major contributor was M-124, the best calculator user and the best student in mathematics in School B.

Excerpt 5 (School A - Rational Numbers):

[Problem under discussion is $86\frac{7}{12} - 38\frac{14}{15}$]

R: Where do we start?

M-115: Use 180 for a common denominator.

R: How did you get that?

M-115: Multiply 12 X 15.

R: Good, so give me the fractions.

[As researcher got these he recorded on the chalkboard]

F-116: $86\frac{105}{180}$ [she multiplied $\underline{7 \times 15 =}$ to provide the 105]

and $36\frac{168}{180}$

[Researcher wrote $86\frac{105}{180} - 36\frac{168}{180}$]

R: Now what?

M-113: Borrow from 86 and that'll give you 285 over 180 [he did the calculation mentally]

R: Good so we now have ....

F-116: $85\frac{285}{180} - 36\frac{168}{180}$

R: So that will be? ....

Several: $47\frac{117}{180}$

[At this point one student requested an explanation of the source of 105 and 168. A friend next to her explained and she seemed satisfied with the explanation. Researcher showed how 285 came about.]

R: Is 117 a prime number?

Several: Yes

M-113: 3 goes [meaning 3 divides 117]

R: Good. Give me a reduced form then for $\frac{117}{180}$.

M-113: 39 over 60 [He had used calculator to get the 39]

[From this step several students suggested that $\frac{39}{60}$ reduces further to $\frac{13}{20}$]

R: If we had realized that 3 divides 117, 39 times and then 3 divides 39, we could have divided right at the beginning by what?

F-111: 6

[Student was corrected on this by several others.]

Excerpt 6 (School B):

[This excerpt illustrates a class discussion and the limited replies of the students. The researcher's questions are usually such that students are led through a problem in small steps.]

[The questions under consideration are as follows:

a) Find the fifth power of 6.

b) Do this in a second way.

c) Do this again in a third way.]

R: What did you do for that one, F-120?

F-120: $6y^x5 =$

R: Right. That's probably the best and simplest method. What's another way for the next question?

M-122: 6 X 6 X 6 X 6 X 6 =

R: Right, that's another way.

M-125: I had a different way.

R: What is that, M-125?

M-125: $6x^2x^2$ X 6 =

R: What does this do [showing $6x^2$]?

M-123: Doubles it.

R: No, 6 + 6 is 12.

M-123: Oh, squares it.

R: Right, so he has 6 X 6 [up to $6x^2$] and when he presses x^2 again what does he have?

M-123: 36 X 6

R: No.

M-123: Well it's squaring the last number.

R: Right but you don't get 36 X 6.

M-125: One thousand two hundred ......

R: [Interrupting] I don't want the actual numbers but an explanation of the power of 6.

M-124: You get 6 X 6 X 6 X 6 .

R: Good. [Researcher reviewed how that comes about] And when he then multiplied by 6 [showing $6x^2x^2$ X 6], he does get the correct answer. So this is a correct way to do it different from the first ones.

What's another way of doing it?

M-123: 6 X K = = = =

R: Right, so we have four ways of getting this answer.

M-122: Shouldn't we go 6 X K6 = for that last one?

R: No, it isn't necessary [explanation followed that the first = sign entry following K will multiply the 6 in display by the 6 as constant].

M-124: Yes, I did it, like, a long way ...

(6 X 6) X (6 X 6) X 6 =

R: Right, and this is really the same as we had before except that it lumps some factors together with parentheses so it's not very different from 6 X 6 X 6 X 6 X 6 X 6 = .

APPENDIX K

SAMPLE WORKSHEET USED IN SCHOOL A
BY A TEACHER DOING SOME CALCULATOR ACTIVITIES

MULTIPLYING DECIMALS
(USING CALCULATORS)

Name ____________________

I. Multiply the following numbers on your calculator. Record your answer.

Group A	Group B
3 X 13 =	2 X 12 =
3 X 1.3 =	2 X 1.2 =
3 X 0.13 =	2 X 0.12 =
0.3 X 13 =	0.2 X 12 =
0.3 X 1.3 =	0.2 X 1.2 =

II. (1) What is the same about all the questions in group A? ____________________

(2) What is the same about all the answers in group A? ____________________

(3) What is different about all the questions in group B? ____________________

(4) What is different about all the answers in group B? ____________________

III. What is the relationship between the number of digits to the right of the decimal point in the question and in the answer? ____________________

____

IV. Write a rule for multiplying numbers involving decimals. ____________________

____

APPENDIX L

INTERVIEW WITH M-118 ON PERIMETER

INTERVIEW WITH M-118 ON PERIMETER

[The first two problems given were: (1) What is the area of a city lot 18.6 m wide and 33.5 m long? and (2) How much would it cost to fence the above lot completely (all four sides) if it costs $13.95 per metre to get this done?]

S: I don't get that question.

R: Why?

S: I don't know how you multiply it and that.

R: Well, how would you find out how much fence you need.

S: Times 13.95

R: We had a lot and we're told it's so wide and so long and we wish to find out the number of metres of fence we'll need to get around this lot.

S: I did the first one and it's ....

R: But let's not get the first one into this. Let's just try to stick with this second problem as if it's a new question.

S: Oh, do you times 18.6?

R: Don't start getting formulas involved. Just picture this lot [used the room as a model of the lot and pointed out all dimensions]. What do I need to know first if I'm going to fence this lot?

S: How big it is?

R: What do you mean how big?

S: All together like 18.6 times 33.5.

R: But why multiplication. What will that give you?

S: The answer [laughing]

R: [Again the room model was used and researcher pointed out distance from one corner to another and called that 18.6 m and so forth around the room]. What could I do to get the total?

S: Measure it.

R: Well, I know the first side is 18.6 m [and so forth around the room, stating dimensions of each side]. What do I do to get the first quantity I need?

S: What do you do to get it? [Laughing]

R: Yes.

S: Measure it.

R: But I've already got the measure for each side. Now what?

S: I don't know. [Laughing]

Another student: Go 18.6 times 2.

R: But even if I didn't multiply by 2, could I not just add all four measures?

[So researcher and student went through stating how all four measures would be added.]

Now we get the total number of metres of fence needed and each metre costs ....?

S: 13.95

R: So?

S: Now you've got me confused. After I plus them altogether what do I do? Do I times them by 2 or multiply?

R: Why multiply them by 2?

S: Well, just multiply by $13.95?

R: Right, because each metre costs that amount.